x6 EXTRA TOPICS RELATED TO Filters and Tuned Amplifiers

This supplement contains material removed from previous editions of the textbook. These topics continue to be relevant and for this reason will be of great value to many instructors and students.

The topics presented here build on and supplement the material presented in Chapter 14 of the eighth edition.

x6.1 First-Order and Second-Order Filter Functions

In this section, we shall study the simplest filter transfer functions, those of first and second order. These functions are useful in their own right in the design of simple filters. First- and second-order filters can also be cascaded to realize a high-order filter. Cascade design is in fact one of the most popular methods for the design of active filters (those utilizing op amps and RC circuits). Because the filter poles occur in complex-conjugate pairs, a high-order transfer function $T(s)$ is factored into the product of second-order functions. If $T(s)$ is odd, there will also be a first-order function in the factorization. Each of the second-order functions [and the first-order function when $T(s)$ is odd] is then realized using an op amp–RC circuit, and the resulting blocks are placed in cascade. If the output of each block is taken at the output terminal of an op amp where the impedance level is low (ideally zero), cascading does not change the transfer functions of the individual blocks. Thus the overall transfer function of the cascade is simply the product of the transfer functions of the individual blocks, which is the original $T(s)$.

x6.1.1 First-Order Filters

The general first-order transfer function is given by

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0} \tag{x6.1}$$

This **bilinear transfer function** characterizes a first-order filter with a pole at $s = -\omega_0$, a transmission zero at $s = -a_0/a_1$, and a high-frequency gain that approaches a_1. The numerator coefficients, a_0 and a_1, determine the type of filter (e.g., low pass, high pass, etc.). Some special cases together with passive (RC) and active (op amp–RC) realizations are shown in Fig. x6.1. Note that the active realizations provide considerably more versatility than their passive counterparts; in many cases the gain can be set to a desired value, and some transfer function parameters can be adjusted without affecting others. The output impedance of the active circuit is also very low, making cascading easily possible. The op amp, however, limits the high-frequency operation of the active circuits.

An important special case of the first-order filter function is the **all-pass filter** shown in Fig. x6.2. Here, the transmission zero and the pole are symmetrically located relative to the $j\omega$ axis. (They are said to display mirror-image symmetry with respect to the $j\omega$ axis.) Observe that although the transmission of the all-pass filter is (ideally) constant at all frequencies, its phase shows frequency selectivity. All-pass filters are used as phase shifters and in systems that require phase shaping (e.g., in the design of circuits called *delay equalizers*, which cause the overall time delay of a transmission system to be constant with frequency).

EXERCISES

xD6.1 Using $R_1 = 10$ kΩ, design the op amp–RC circuit of Fig. x6.1(b) to realize a high-pass filter with a corner frequency of 10^4 rad/s and a high-frequency gain of 10.

Ans. $R_2 = 100$ kΩ; $C = 0.01$ μF

xD6.2 Design the op amp–RC circuit of Fig. x6.2 to realize an all-pass filter with a 90° phase shift at 10^3 rad/s. Select suitable component values.

Ans. Possible choices: $R = R_1 = R_2 = 10$ kΩ; $C = 0.1$ μF

x6.1.2 Second-Order Filter Functions

The general second-order (or **biquadratic**) filter transfer function is usually expressed in the standard form

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0/Q)s + \omega_0^2} \tag{x6.1}$$

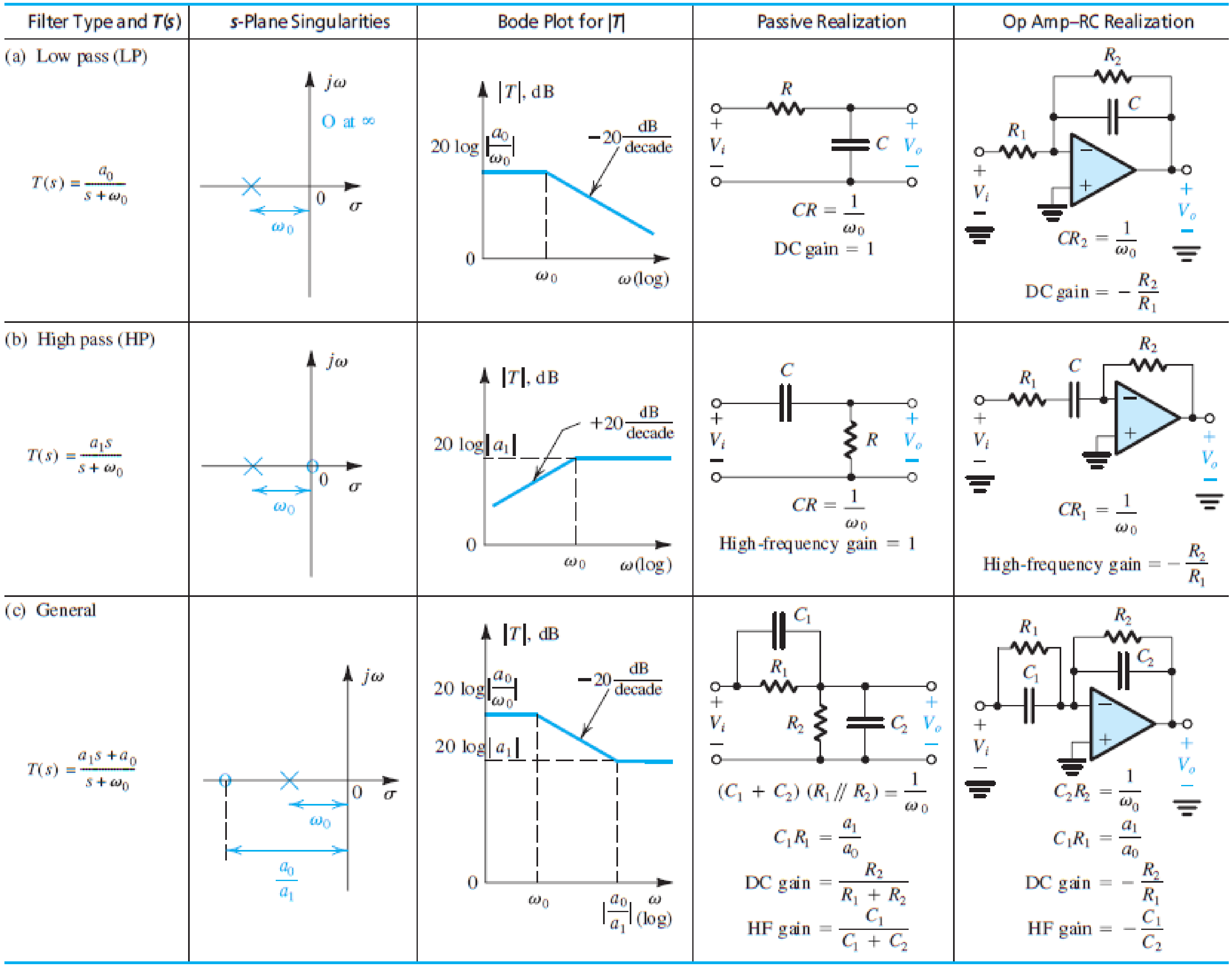


Figure x6.1 First-order filters.

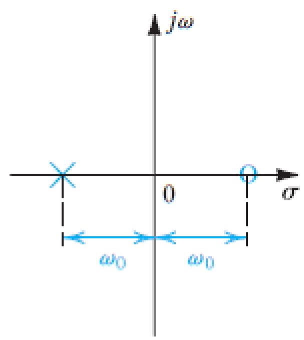
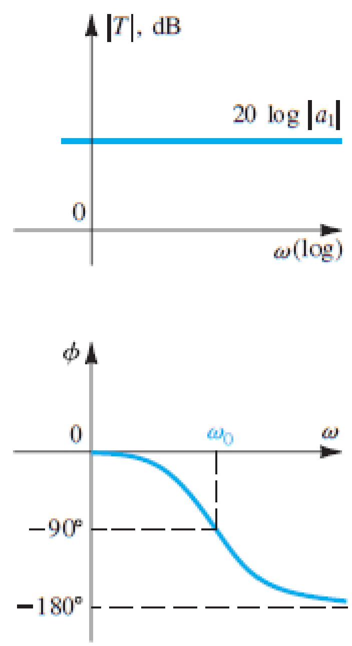
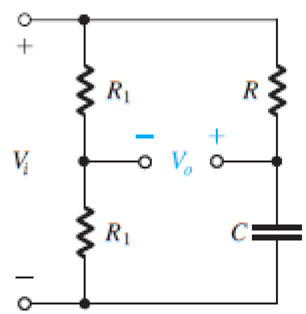
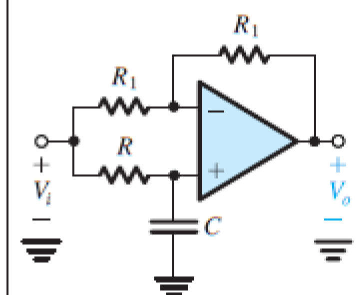
$T(s)$	Singularities	$\|T\|$ and ϕ	Passive Realization	Op Amp–RC Realization
All pass (AP) $T(s) = -a_1\frac{s-\omega_0}{s+\omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$ $\left\|\frac{V_o}{V_i}\right\| = 1$ $\phi(\omega) = -2\tan^{-1}(\omega CR)$

Figure x6.2 First-order all-pass filter.

where ω_0 and Q determine the natural modes (poles) according to

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-(1/4Q^2)} \tag{x6.3}$$

We are usually interested in the case of complex-conjugate natural modes, obtained for $Q > 0.5$. Figure x6.3 shows the location of the pair of complex-conjugate poles in the s plane. Observe that the radial distance of the natural modes (from the origin) is equal to ω_0, which is known as the **pole frequency**. The parameter Q determines the distance of the poles from the $j\omega$ axis: the higher the value of Q, the closer the poles are to the $j\omega$ axis, and the more selective the filter response becomes. An infinite value for Q locates the poles on the $j\omega$ axis and can yield sustained oscillations in the circuit realization. A negative value of Q implies that the poles are in the right half of the s plane, which certainly produces oscillations. The parameter Q is called the **pole quality factor**, or simply **pole *Q***.

The transmission zeros of the second-order filter are determined by the numerator coefficients, a_0, a_1, and a_2. It follows that the numerator coefficients determine the type of second-order filter function (i.e., LP, HP, etc.). Seven special cases of interest are illustrated in Fig. x6.4. For each case we give the transfer function, the s-plane locations of the transfer function singularities, and the magnitude response. Circuit realizations for the various second-order filter functions are given in Chapter 14 and some parts of this supplement.

All seven special second-order filters have a pair of complex-conjugate natural modes characterized by a frequency ω_0 and a quality factor Q.

In the low-pass (LP) case, shown in Fig. x6.4(a), the two transmission zeros are at $s = \infty$. The magnitude response can exhibit a peak with the details indicated. It can be shown that the peak occurs only for $Q > 1/\sqrt{2}$. The response obtained for $Q = 1/\sqrt{2}$ is the Butterworth, or maximally flat, response.

The high-pass (HP) function shown in Fig. x6.4(b) has both transmission zeros at $s = 0$ (dc). The magnitude response shows a peak for $Q > 1/\sqrt{2}$, with the details of the response as indicated. Observe the duality between the LP and HP responses.

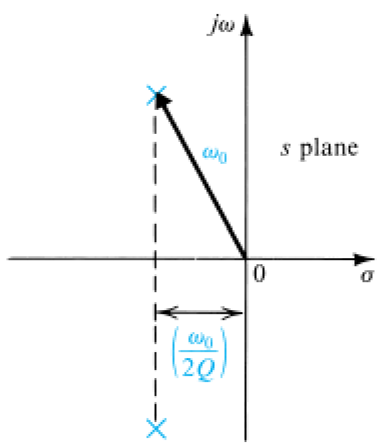


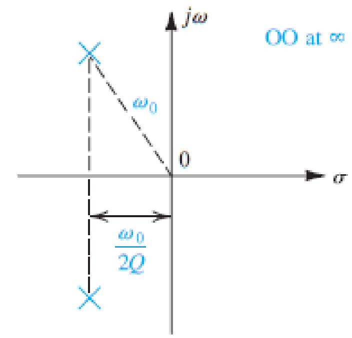
Figure x6.3 Definition of the parameters ω_0 and Q of a pair of complex-conjugate poles.

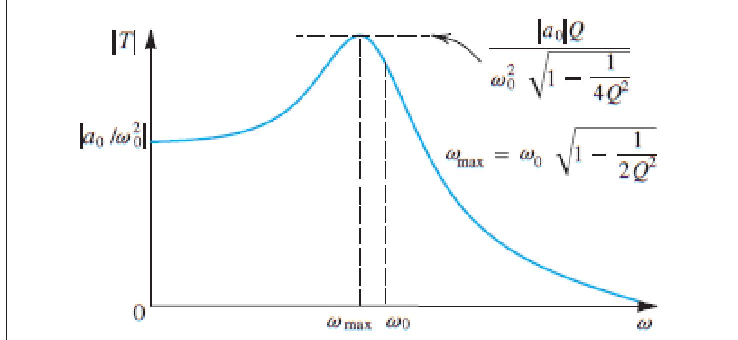
Filter Type and $T(s)$	s-Plane Singularities	$\|T\|$

(a) Low pass (LP)

$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\text{DC gain} = \frac{a_0}{\omega_0^2}$$

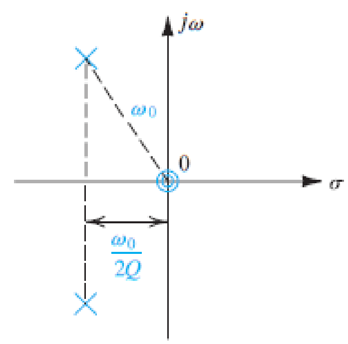


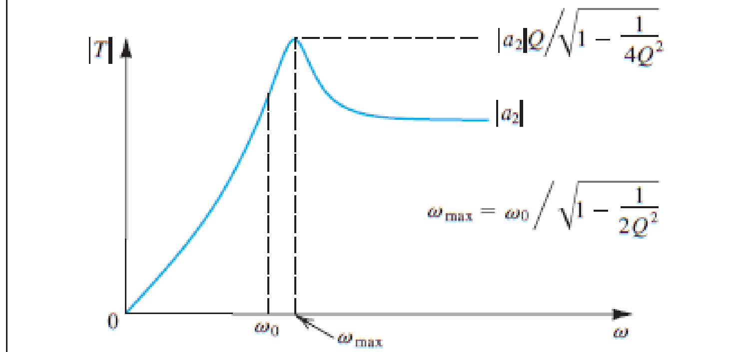


(b) High pass (HP)

$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\text{High-frequency gain} = a_2$$

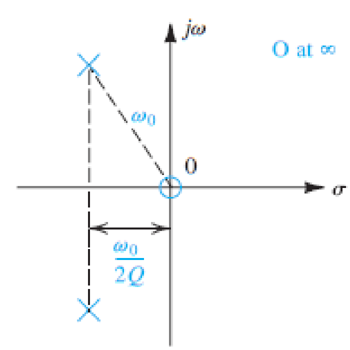




(c) Bandpass (BP)

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\text{Center-frequency gain} = \frac{a_1 Q}{\omega_0}$$



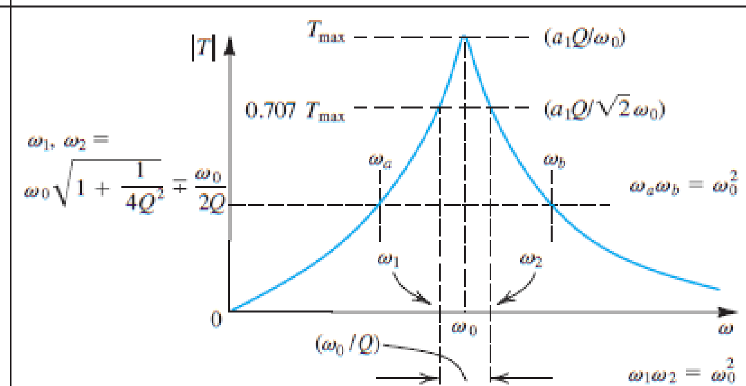


Figure x6.4 Second-order filtering functions.

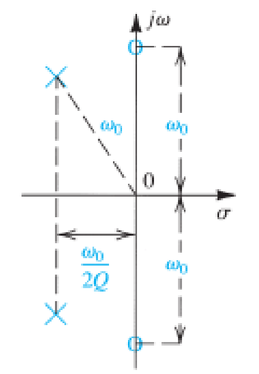
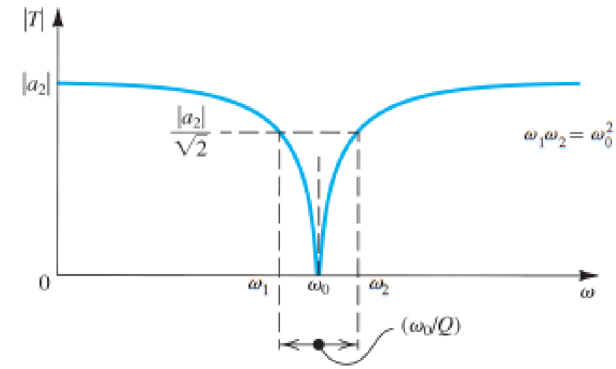
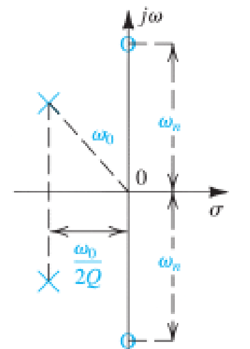
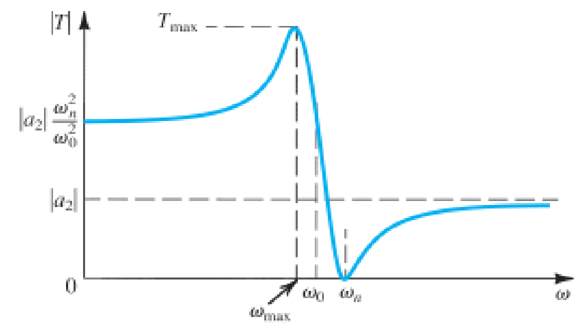
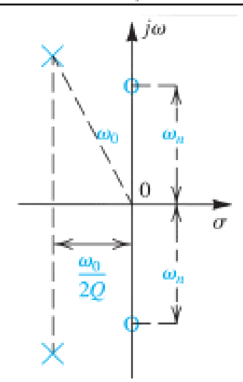
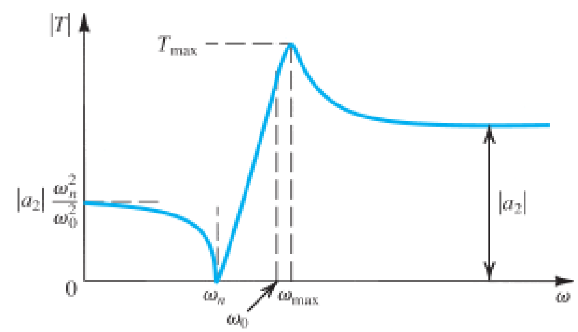
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T \rvert$
(d) Notch $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain = High-frequency gain $= a_2$		
(e) Low-pass notch (LPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \geq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		 $\omega_{max} = \omega_0 \sqrt{\dfrac{\left(\dfrac{\omega_n^2}{\omega_0^2}\right)\left(1 - \dfrac{1}{2Q^2}\right) - 1}{\dfrac{\omega_n^2}{\omega_0^2} + \dfrac{1}{2Q^2} - 1}}$
(f) High-pass notch (HPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \leq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		 $T_{max} = \dfrac{\lvert a_2 \rvert \, \lvert \omega_n^2 - \omega_{max}^2 \rvert}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{max}^2}}$

Figure x6.4 *continued*

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain = a_2

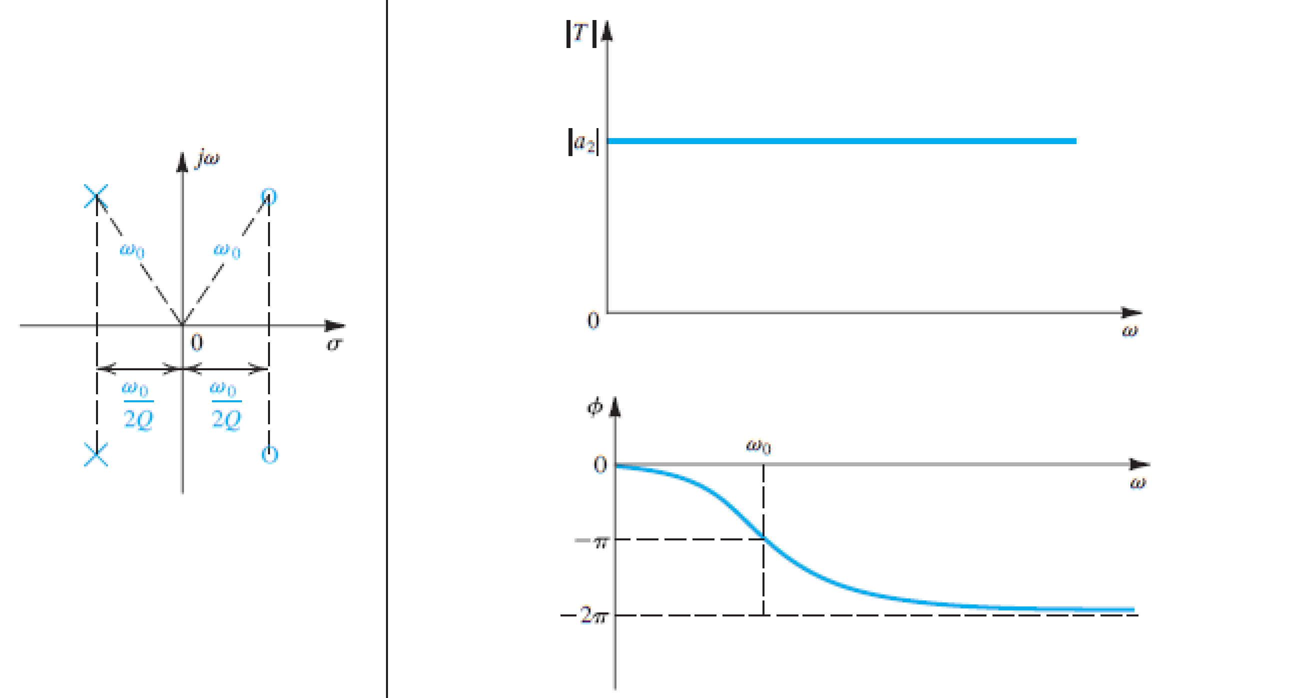


Figure x6.4 *continued*

Next consider the bandpass (BP) filter function shown in Fig. x6.4(c). Here, one transmission zero is at $s = 0$ (dc), and the other is at $s = \infty$. The magnitude response peaks at $\omega = \omega_0$. Thus the **center frequency** of the bandpass filter is equal to the pole frequency ω_0. The selectivity of the second-order bandpass filter is usually measured by its *3-dB bandwidth*. This is the difference between the two frequencies ω_1 and ω_2 at which the magnitude response is 3 dB below its maximum value (at ω_0). It can be shown that

$$\omega_1, \omega_2 = \omega_0\sqrt{1 + (1/4Q^2)} \pm \frac{\omega_0}{2Q} \tag{x6.4}$$

Thus,

$$BW \equiv \omega_2 - \omega_1 = \omega_0/Q \tag{x6.1}$$

Observe that as Q increases, the bandwidth decreases and the bandpass filter becomes more selective.

If the transmission zeros are located on the jω axis, at the complex-conjugate locations $\pm j\omega_n$, then the magnitude response exhibits zero transmission at $\omega = \omega_n$. Thus a notch in the magnitude response occurs at $\omega = \omega_n$, and ω_n is known as the notch frequency. Three cases of the second-order notch filter are possible: the regular notch, obtained when $\omega_n = \omega_0$ [Fig. x6.4(d)]; the low-pass notch, obtained when $\omega_n > \omega_0$ [Fig. x6.4(e)]; and the high-pass notch, obtained when $\omega_n < \omega_0$ [Fig. x6.4(f)]. The reader is urged to verify the response details given in these figures (a rather tedious task, though!). Observe that in all notch cases, the transmission at dc and at $s = \infty$ is finite. This is so because there are no transmission zeros at either $s = 0$ or $s = \infty$.

The last special case of interest is the all-pass (AP) filter whose characteristics are illustrated in Fig. x6.4(g). Here the two transmission zeros are in the right half of the s plane, at the mirror-image locations of the poles. (This is the case for all-pass functions of any order.) The magnitude response of the all-pass function is constant over all frequencies; the flat gain, as it is called, is in our case equal to $|a_2|$. The frequency selectivity of the all-pass function is in its phase response.

EXERCISES

x6.3 For a maximally flat second-order low-pass filter ($Q = 1/\sqrt{2}$), show that at $\omega = \omega_0$ the magnitude response is 3 dB below the value at dc.

x6.4 Give the transfer function of a second-order bandpass filter with a center frequency of 10^5 rad/s, a center-frequency gain of 10, and a 3-dB bandwidth of 10^3 rad/s.

Ans.

$$T(s) = \frac{10^4 s}{s^2 + 10^3 s + 10^{10}}$$

x6.5 (a) For the second-order notch function with $\omega = \omega_0$, show that for the attenuation to be greater than A dB over a frequency band BW_a, the value of Q is given by

$$Q \leq \frac{\omega_0}{BW_a\sqrt{10^{A/10} - 1}}$$

(*Hint:* First, show that any two frequencies, ω_1 and ω_2, at which $|T|$ is the same are related by $\omega_1\omega_2 = \omega_0^2$.)

(b) Use the result of (a) to show that the 3-dB bandwidth is ω_0/Q, as indicated in Fig. x6.4(d).

x6.6 Consider a low-pass notch with $\omega_0 = 1$ rad/s, $Q = 10$, $\omega_n = 1.2$ rad/s, and a dc gain of unity. Find the frequency and magnitude of the transmission peak. Also find the high-frequency transmission.

Ans. 0.986 rad/s; 3.17; 0.69

x6.2 Single-Amplifier Biquadratic Active Filters

The op amp–RC biquadratic circuits studied in Sections 14.5 and 14.6 of the eighth edition provide good performance, are versatile, and are easy to design and to adjust (tune) after final assembly. Unfortunately, however, they are not economic in their use of op amps, requiring three or four amplifiers per second-order section. This can be a problem, especially in applications that call for conservation of power-supply current: for instance, in a battery-operated instrument. In this section we shall study a class of second-order filter circuits that requires only one op amp per biquad. These minimal realizations, however, suffer a greater dependence on the limited gain and bandwidth of the op amp and can also be more sensitive to the unavoidable tolerances in the values of resistors and capacitors than the multiple-op-amp biquads. The **single-amplifier biquads** (SABs) are therefore limited to the less stringent filter specifications—for example, pole Q factors less than about 10.

The synthesis of SAB circuits is based on the use of feedback to move the poles of an RC circuit from the negative real axis, where they naturally lie, to the complex-conjugate locations required to provide selective filter response. The synthesis of SABs follows a two-step process:

1. Synthesis of a feedback loop that realizes a pair of complex-conjugate poles characterized by a frequency ω_0 and a Q factor Q.
2. Injecting the input signal in a way that realizes the desired transmission zeros.

x6.2.1 Synthesis of the Feedback Loop

Consider the circuit shown in Fig. x6.5(a), which consists of a two-port RC network n placed in the negative-feedback path of an op amp. We shall assume that, except for having a finite gain A, the op amp is ideal. We shall denote by $t(s)$ the open-circuit voltage-transfer function of the RC network n, where the definition of $t(s)$ is illustrated in Fig. x6.5(b). The transfer function $t(s)$ can in general be written as the ratio of two polynomials $N(s)$ and $D(s)$:

$$t(s) = \frac{N(s)}{D(s)}$$

The roots of $N(s)$ are the transmission zeros of the RC network, and the roots of $D(s)$ are its poles. Study of circuit theory shows that while the poles of an RC network are restricted to lie on the negative real axis, the zeros can in general lie anywhere in the s plane.

The loop gain $L(s)$ of the feedback circuit in Fig. x6.5(a) can be determined using the method of Section 11.3.3 of the textbook. It is simply the product of the op-amp gain A and the transfer function $t(s)$,

$$L(s) = At(s) = \frac{AN(s)}{D(s)} \quad \text{(x6.6)}$$

Substituting for $L(s)$ into the characteristic equation

$$1 + L(s) = 0 \quad \text{(x6.7)}$$

results in the poles s_p of the closed-loop circuit obtained as solutions to the equation

$$(s_p) = -\frac{1}{A} \quad \text{(x6.8)}$$

In the ideal case, $A = \infty$ and the poles are obtained from

$$N(s_p) = 0 \quad \text{(x6.9)}$$

That is, *the filter poles are identical to the zeros of the RC network.*

Since our objective is to realize a pair of complex-conjugate poles, we should select an RC network that can have complex-conjugate transmission zeros. The simplest such networks are the bridged-T networks shown in Fig. x6.6 together with their transfer functions $t(s)$ from b to a, with a open-circuited. As an example, consider the circuit generated by placing the bridged-T network of Fig. x6.6(a) in the negative-feedback path of an op amp, as shown in Fig. x6.7. The pole polynomial of the active-filter circuit will be equal to the numerator polynomial of the bridged-T network; thus,

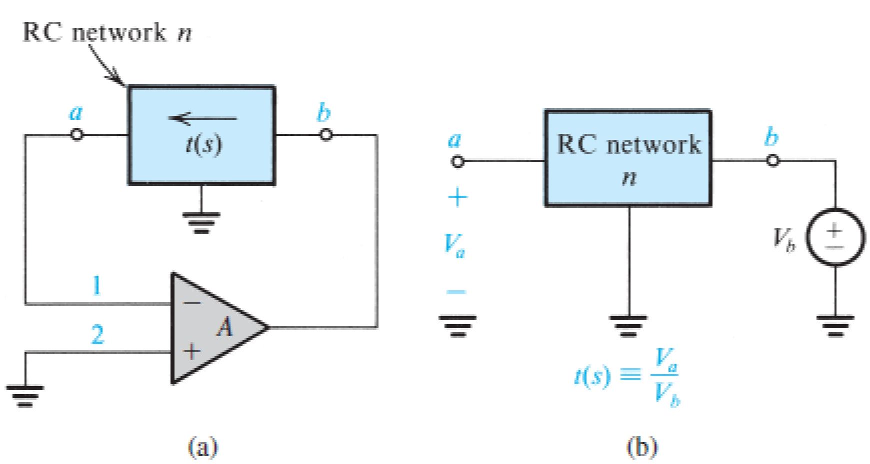


Figure x6.5 **(a)** Feedback loop obtained by placing a two-port RC network n in the feedback path of an op amp. **(b)** Definition of the open-circuit transfer function $t(s)$ of the RC network.

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}$$

which enables us to obtain ω_0 and Q as

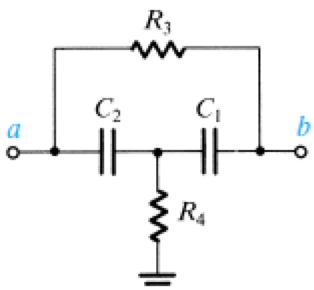
$$\omega_0 = \frac{1}{\sqrt{C_1C_2R_3R_4}} \tag{x6.10}$$

$$Q = \left[\frac{\sqrt{C_1C_2R_3R_4}}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{-1} \tag{x6.11}$$

If we are designing this circuit, ω_0 and Q are given and Eqs. (x6.10) and (x6.11) can be used to determine C_1, C_2, R_3, and R_4. It follows that there are two degrees of freedom. Let us use one of these by selecting $C_1 = C_2 = C$. Let us also denote $R_3 = R$ and $R_4 = R/m$. By substituting in Eqs. (x6.10) and (x6.11), and with some manipulation, we obtain

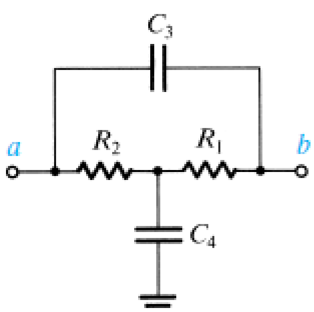
$$m = 4Q^2 \tag{x6.12}$$

$$CR = \frac{2Q}{\omega_0} \tag{x6.13}$$



$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4}}$$

(a)



$$t(s) = \frac{s^2 + s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_4} + \frac{1}{C_3C_4R_1R_2}}{s^2 + s\left(\frac{1}{C_4R_1} + \frac{1}{C_4R_2} + \frac{1}{C_3R_2}\right) + \frac{1}{C_3C_4R_1R_2}}$$

(b)

Figure x6.6 Two RC networks (called bridged-T networks) that can have complex transmission zeros. The transfer functions given are from b to a, with a open-circuited.

Thus if we are given the value of Q, Eq. (x6.12) can be used to determine the ratio of the two resistances R_3 and R_4. Then the given values of ω_0 and Q can be substituted in Eq. (x6.13) to determine the time constant CR. There remains one degree of freedom—the value of C or R can be arbitrarily chosen. In an actual design, this value, which sets the *impedance level* of the circuit, should be chosen so that the resulting component values are practical.

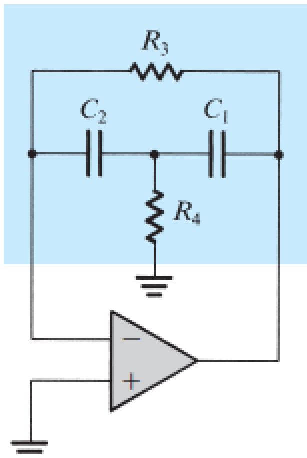


Figure x6.7 An active-filter feedback loop generated using the bridged-T network of Fig. x6.6(a).

EXERCISES

xD6.7 Design the circuit of Fig. x6.7 to realize a pair of poles with $\omega_0 = 10^4$ rad/s and $Q = 1$. Select $C_1 = C_2 = 1$ nF.

Ans. $R_3 = 200$ kΩ; $R_4 = 50$ kΩ

x6.8 For the circuit designed in Exercise x6.7, find the location of the poles of the RC network in the feedback loop.

Ans. -0.382×10^4 and -2.618×10^4 rad/s

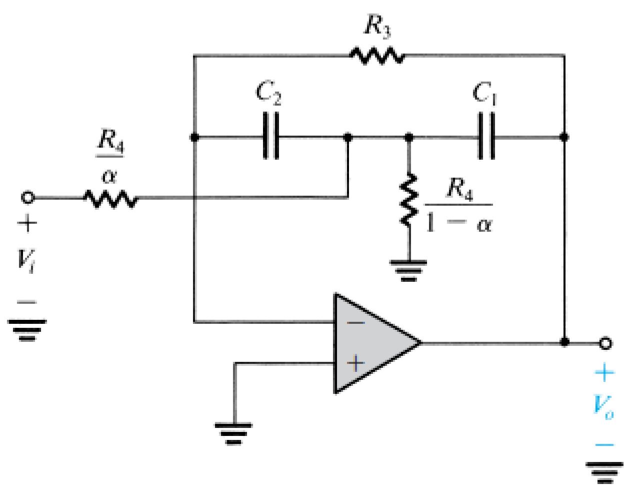
x6.2.2 Injecting the Input Signal

Having synthesized a feedback loop that realizes a given pair of poles, we now consider connecting the input signal source to the circuit. We wish to do this, of course, without altering the poles.

Since, for the purpose of finding the poles of a circuit, an ideal voltage source is equivalent to a short circuit, it follows that any circuit node that is connected to ground can instead be connected to the input voltage source without causing the poles to change. Thus the method of injecting the input voltage signal into the feedback loop is simply to disconnect a component (or several components) that is (are) connected to ground and connect it (them) to the input source. Depending on the component(s) through which the input signal is injected, different transmission zeros are obtained. This is, of course, the same method we used in Section 14.4 with the LCR resonator and in Section 14.5 with the biquads based on the LCR resonator.

As an example, consider the feedback loop of Fig. x6.7. Here we have two grounded nodes (one terminal of R_4 and the positive input terminal of the op amp) that can serve for injecting the input signal. Figure x6.8(a) shows the circuit with the input signal injected through part of the resistance R_4. Note that the two resistances R_4/α and $R_4/(1-\alpha)$ have a parallel equivalent of R_4.

Analysis of the circuit to determine its voltage-transfer function $T(s) \equiv V_o(s)/V_i(s)$ is illustrated in Fig. x6.8(b). Note that we have assumed the op amp to be ideal and have indicated the order of the analysis steps by the circled numbers. The final step, number 9, consists of writing a node equation at X and substituting for V_x by the value determined in step 5. The result is the transfer function



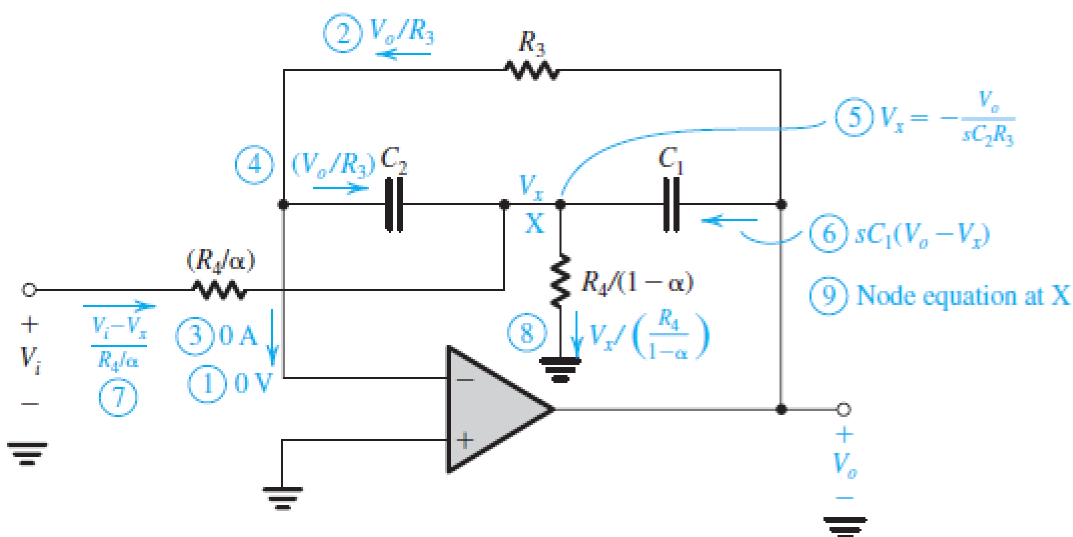


Figure x6.8 **(a)** The feedback loop of Fig. x6.7 with the input signal injected through part of resistance R_4. This circuit realizes the bandpass function. **(b)** Analysis of the circuit in (a) to determine its voltage transfer function $T(s)$ with the order of the analysis steps indicated by the circled numbers.

$$\frac{V_o}{V_i} = \frac{-s(\alpha/C_1R_4)}{s^2 + s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}$$

We recognize this as a bandpass function whose center-frequency gain can be controlled by the value of α. As expected, the denominator polynomial is identical to the numerator polynomial of $t(s)$ given in Fig. x6.6(a).

EXERCISE

x6.9 Use the component values obtained in Exercise x6.7 to design the bandpass circuit of Fig. x6.8(a). Determine the values of (R_4/α) and $R_4/(1-\alpha)$ to obtain a center-frequency gain of unity.

Ans. 100 kΩ; R_4 = 50 kΩ

x6.2.3 Generating Equivalent Feedback Loops

The **complementary transformation** of feedback loops is based on the property of linear networks illustrated in Fig. x6.9 for the two-port (three-terminal) network *n*. In Fig. x6.9(a), terminal *c* is grounded and a signal V_b is applied to terminal *b*. The transfer function from *b* to *a* with *c* grounded is denoted *t*. Then, in Fig. x6.9(b), terminal *b* is grounded and the input signal is applied to terminal *c*. The transfer function from *c* to *a* with *b* grounded can be shown to be the complement of *t*—that is, $1-t$.

Applying the complementary transformation to a feedback loop to generate an equivalent feedback loop is a two-step process:

1. Nodes of the feedback network and any of the op-amp inputs that are connected to ground should be disconnected from ground and connected to the op-amp output. Conversely, those nodes that were connected to the op-amp output should be now connected to ground. That is, we simply interchange the op-amp output terminal with ground.

2. The two input terminals of the op amp should be interchanged.

The feedback loop generated by this transformation has the same characteristic equation, and hence the same poles, as the original loop.

To illustrate, we show in Fig. x6.10(a) the feedback loop formed by connecting a two-port RC network in the negative-feedback path of an op amp. Application of the complementary transformation to this loop results in the feedback loop of Fig. x6.10(b). Note that in the latter loop the op amp is used in the unity-gain follower configuration. We shall now show that the two loops of Fig. x6.10 are equivalent.

If the op amp has an open-loop gain *A*, the follower in the circuit of Fig. x6.10(b) will have a gain of $A/(A+1)$. This, together with the fact that the transfer function of network *n* from *c* to *a* is $1-t$ (see Fig. x6.9), enables us to write for the circuit in Fig. x6.10(b) the characteristic equation

$$1 - \frac{A}{A+1}(1-t) = 0$$

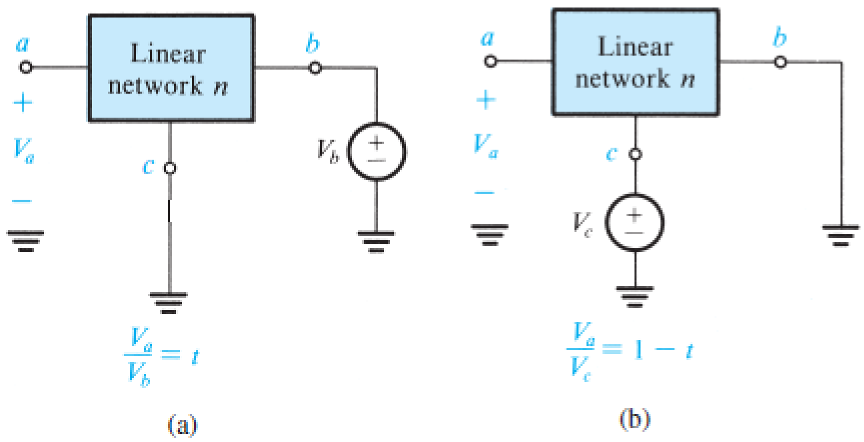


Figure x6.9 Interchanging input and ground results in the complement of the transfer function.

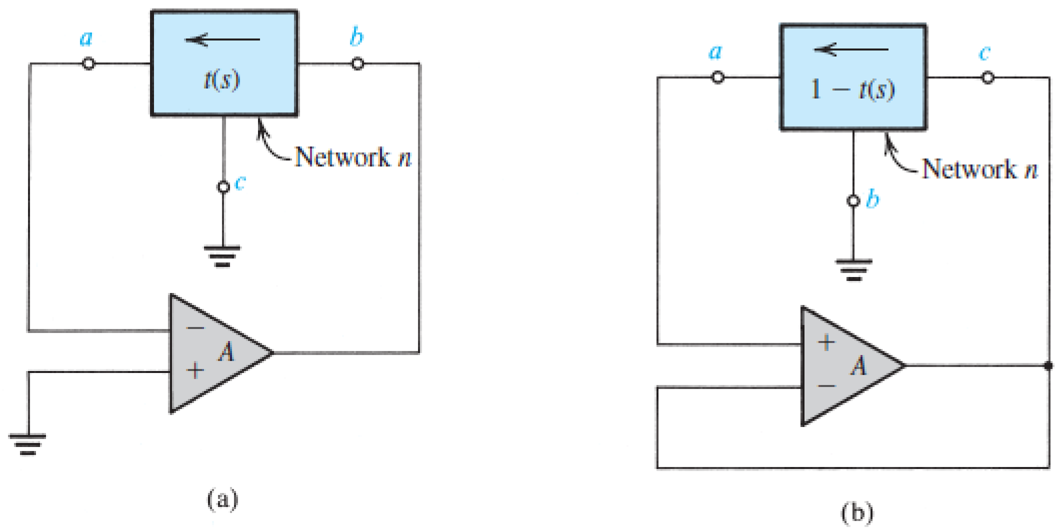


Figure x6.10 Application of the complementary transformation to the feedback loop in **(a)** results in the equivalent loop (same poles) shown in **(b)**.

This equation can be manipulated to the form

$$1 + At = 0$$

which is the characteristic equation of the loop in Fig. x6.10(a). As an example, consider the application of the complementary transformation to the feedback loop of Fig. x6.7: The feedback loop of Fig. x6.11(a) results. Injecting the input signal through C_1 results in the circuit in Fig. x6.11(b), which can be shown (by direct analysis) to realize a second-order high-pass function. This circuit is one of a family of SABs known as the **Sallen-and-Key circuits**, after their originators. The design of the circuit in Fig. x6.11(b) is based on Eqs. (x6.10) through (x6.13): namely, $R_3 = R$, $R_4 = R/4Q^2$, $C_1 = C_2 = C$, $CR = 2Q/\omega_0$, and the value of C is arbitrarily chosen to be practically convenient.

As another example, Fig. x6.12(a) shows the feedback loop generated by placing the two-port RC network of Fig. x6.6(b) in the negative-feedback path of an op amp. For an ideal op amp, this feedback loop realizes a pair of complex-conjugate natural modes having the same location as the zeros of $t(s)$ of the RC network. Thus, using the expression for $t(s)$ given in Fig. x6.6(b), we can write for the active-filter poles

$$\omega_0 = 1/\sqrt{C_3 C_4 R_1 R_2} \quad \text{(x6.14)}$$

$$= \left[\frac{\sqrt{C_3 C_4 R_1 R_2}}{C_4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]^{-1} \quad \text{(x6.15)}$$

Normally the design of this circuit is based on selecting $R_1 = R_2 = R$, $C_4 = C$, and $C_3 = C/m$. When substituted in Eqs. (x6.14) and (x6.15), these yield

$$m = 4Q^2 \quad \text{(x6.16)}$$

$$CR = 2Q/\omega_0 \quad \text{(x6.17)}$$

with the remaining degree of freedom (the value of C or R) left to the designer to choose.

Injecting the input signal to the C_4 terminal that is connected to ground can be shown to result in a bandpass realization. If, however, we apply the complementary transformation to the feedback loop in Fig. x6.12(a), we obtain the equivalent loop in Fig. x6.12(b). The loop equivalence means that the circuit of Fig. x6.12(b) has the same poles and thus the same ω_0 and Q and the same design equations (Eqs. x6.14 through x6.17). The new loop in Fig. x6.12(b) can be used to realize a low-pass function by injecting the input signal as shown in Fig. x6.12(c).

In conclusion, we note that complementary transformation is a powerful tool that enables us to obtain new filter circuits from ones we already have, thus increasing our repertoire of filter realizations.

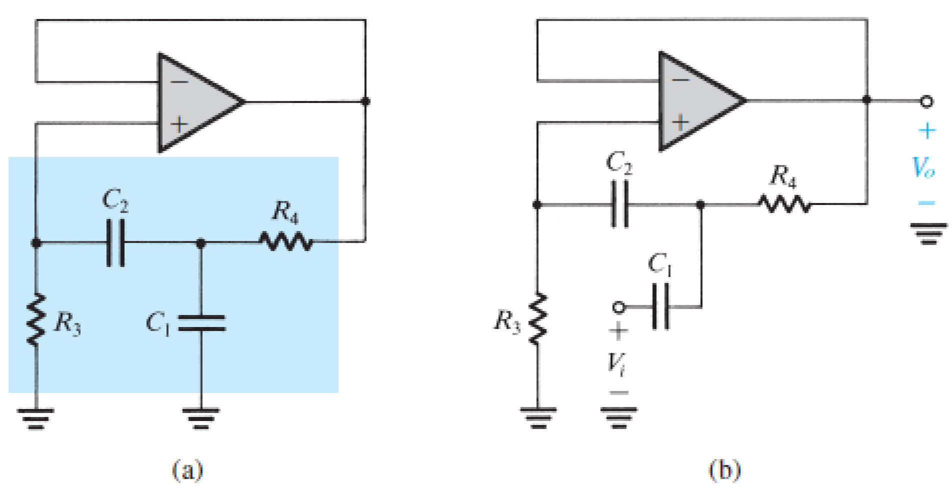


Figure x6.11 **(a)** Feedback loop obtained by applying the complementary transformation to the loop in Fig. x6.7. **(b)** Injecting the input signal through C_1 realizes the high-pass function. This is one of the Sallen-and-Key family of circuits.

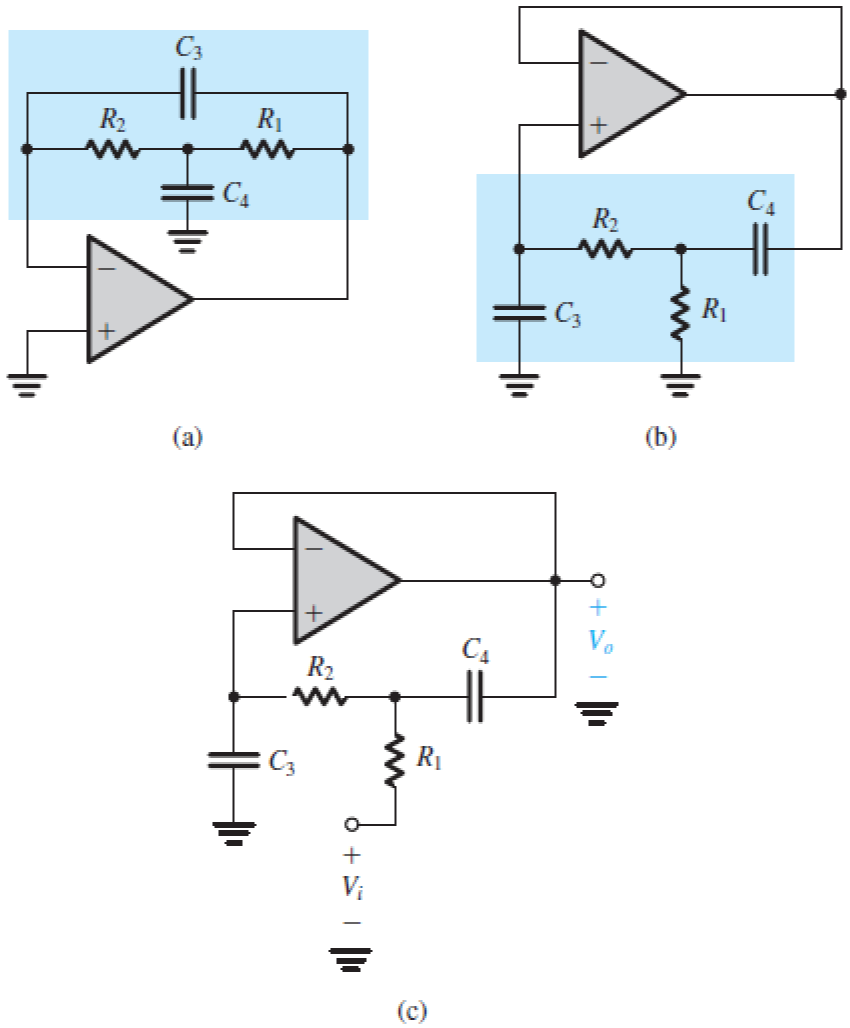


Figure x6.12 **(a)** Feedback loop obtained by placing the bridged-T network of Fig. x6.6(b) in the negative-feedback path of an op amp. **(b)** Equivalent feedback loop generated by applying the complementary transformation to the loop in (a). **(c)** A low-pass filter obtained by injecting V_i through R_1 into the loop in (b).

EXERCISES

x6.10 Analyze the circuit in Fig. x6.12(c) to determine its transfer function $V_o(s)/V_i(s)$ and thus show that ω_0 and Q are indeed those in Eqs. (x6.14) and (x6.15). Also show that the dc gain is unity.

xD6.11 Design the circuit in Fig. x6.12(c) to realize a low-pass filter with $f_0 = 4$ kHz and $Q = 1/\sqrt{2}$. Use 10-kΩ resistors.

Ans. $R_1 = R_2 = 10$ kΩ; $C_3 = 2.81$ nF; $C_4 = 5.63$ nF

x6.3 Sensitivity

Because of the tolerances in component values and because of the finite op-amp gain, the response of the actual assembled filter will deviate from the ideal response. As a means for predicting such deviations, the filter designer employs the concept of **sensitivity**. Specifically, for second-order filters one is usually interested in finding how *sensitive* their poles are relative to variations (both initial tolerances and future drifts) in RC component values and amplifier gain. These sensitivities can be quantified using the **classical sensitivity function** S_x^y, defined as

$$S_x^y \equiv \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} \tag{x6.18}$$

Thus,

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} \tag{x6.19}$$

Here, x denotes the value of a component (a resistor, a capacitor, or an amplifier gain) and y denotes a circuit parameter of interest (say, ω_0 or Q). For small changes

$$S_x^y \simeq \frac{\Delta y / y}{\Delta x / x} \tag{x6.20}$$

Thus we can use the value of S_x^y to determine the per-unit change in y due to a given per-unit change in x. For instance, if the sensitivity of Q relative to a particular resistance R_1 is 5, then a 1% increase in R_1 results in a 5% increase in the value of Q.

Example x6.1

For the feedback loop of Fig. x6.7, find the sensitivities of ω_0 and Q relative to all the passive components and the op-amp gain. Evaluate these sensitivities for the design considered in the preceding section for which $C_1 = C_2$.

Solution

To find the sensitivities with respect to the passive components, called **passive sensitivities**, we assume that the op-amp gain is infinite. In this case, ω_0 and Q are given by Eqs. (x6.10) and (x6.11). Thus for ω_0 we have

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$$

which can be used together with the sensitivity definition of Eq. (x6.19) to obtain

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_4}^{\omega_0} = -\frac{1}{2}$$

For Q we have

$$Q = \left[\sqrt{C_1 C_2 R_3 R_4}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3}\right]^{-1}$$

to which we apply the sensitivity definition to obtain

$$S_{C_1}^{Q} = \frac{1}{2}\left(\sqrt{\frac{C_2}{C_1}} - \sqrt{\frac{C_1}{C_2}}\right)\left(\sqrt{\frac{C_2}{C_1}} + \sqrt{\frac{C_1}{C_2}}\right)^{-1}$$

For the design with $C_1 = C_2$ we see that $S_{C_1}^{Q} = 0$. Similarly, we can show that

$$S_{C_2}^{Q} = 0, \quad S_{R_3}^{Q} = \frac{1}{2}, \quad S_{R_4}^{Q} = -\frac{1}{2}$$

It is important to remember that the sensitivity expression should be derived *before* values corresponding to a particular design are substituted.

Next we consider the sensitivities relative to the amplifier gain. If we assume the op amp to have a finite gain A, the characteristic equation for the loop becomes

$$1 + At(s) = 0 \qquad \text{(x6.21)}$$

where $t(s)$ is given in Fig. x6.6(a). To simplify matters we can substitute for the passive components by their design values. This causes no errors in evaluating sensitivities, since we are now finding the sensitivity with respect to the amplifier gain. Using the design values obtained earlier—namely, $C_1 = C_2 = C$, $R_3 = R$, $R_4 = R/4Q^2$, and $CR = 2Q/\omega_0$—we get

$$t(s) = \frac{s^2 + s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q)(2Q^2 + 1) + \omega_0^2} \qquad \text{(x6.22)}$$

where ω_0 and Q denote the nominal or design values of the pole frequency and Q factor. The actual values are obtained by substituting for $t(s)$ in Eq. (x6.21):

$$s^2 + s\frac{\omega_0}{Q}(2Q^2 + 1) + \omega_0^2 + A\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right) = 0 \qquad \text{(x6.23)}$$

Assuming the gain A to be real and dividing both sides by $(A + 1)$, we get

$$s^2 + s\frac{\omega_0}{Q}\left(1 + \frac{2Q^2}{A + 1}\right) + \omega_0^2 = 0 \qquad \text{(x6.24)}$$

From this equation we see that the actual pole frequency, ω_{0a}, and the pole Q, Q_a, are

$$\omega_{0a} = \omega_0 \qquad \text{(x6.25)}$$

$$Q_a = \frac{Q}{1 + 2A^2/(A + 1)} \qquad \text{(x6.26)}$$

Thus

$$S_A^{\omega_{0a}} = 0$$

$$S_A^{Q_a} = \frac{A}{A+1}\frac{2Q^2/(A+1)}{1+2Q^2/(A+1)}$$

For $A \gg 2Q^2$ and $A \gg 1$ we obtain

$$S_A^{Q_a} \simeq \frac{2Q^2}{A}$$

It is usual to drop the subscript *a* in this expression and write

$$S_A^{Q} \simeq \frac{2Q^2}{A} \qquad \text{(x6.27)}$$

Note that if Q is high ($Q \geq 5$), its sensitivity relative to the amplifier gain can be quite high.

The results of Example x6.1 indicate a serious disadvantage of single-amplifier biquads—the sensitivity of Q relative to the amplifier gain is quite high. Although a technique exists for reducing S_A^Q in SABs (see Sedra et al., 1980), this is done at the expense of increased passive sensitivities. Nevertheless, the resulting SABs are used extensively in many applications. However, for filters with Q factors greater than about 10, one usually opts for one of the multiamplifier biquads studied in Sections 14.5 and 14.6 of the eighth edition of the textbook. For these circuits S_A^Q is proportional to Q, rather than to Q^2 as in the SAB case (Eq. x6.27).

EXERCISE

x6.12 In a particular filter utilizing the feedback loop of Fig. x6.7, with $C_1 = C_2$, use the results of Example x6.1 to find the expected percentage change in ω_0 and Q under the conditions that (a) R_3 is 2% high, (b) R_4 is 2% high, (c) both R_3 and R_4 are 2% high, and (d) both capacitors are 2% low and both resistors are 2% high.

Ans. (a) –1%, +1%; (b) –1%, –1%; (c) –2%, 0%; (d) 0%, 0%

x6.4 Transconductance-C Filters

The op amp–RC circuits studied in Sections 14.2, 14.5, and 14.6 of the textbook and Section x6.3 of this supplement are ideally suited for implementing audio-frequency filters using discrete op amps, resistors, and capacitors, assembled on printed-circuit boards. Such circuits have also been implemented in hybrid thin- or thick-film forms where the op amps are used in chip form (i.e., without their packages).

The limitation of op amp–RC filters to low-frequency applications is a result of the relatively low bandwidth of general-purpose op amps. The lack of suitability of these filter circuits for implementation in IC form stems from:

1. The need for large-valued capacitors, which would require impractically large chip areas;
2. The need for very precise values of RC time constants. This is impossible to achieve on an IC without resorting to expensive trimming and tuning techniques; and
3. The need for op amps that can drive resistive and large capacitive loads. As we have seen, CMOS op amps are usually capable of driving only small capacitances.

x6.4.1 Methods for IC Filter Implementation

We now introduce the three approaches currently in use for implementing filters in monolithic form.

Transconductance-C Filters These utilize transconductance amplifiers or simply transconductors together with capacitors and are hence called G_m–C filters. Because high-quality and high-frequency transconductors can be easily realized in CMOS technology, where small-valued capacitors are plentiful, this filter-design method is very popular at this time. It has been used at medium and high frequencies approaching the hundreds of megahertz range. We shall study this method briefly in this section.

MOSFET-C Filters These use the two-integrator-loop circuits of Section 14.5 of the textbook but with the resistors replaced with MOSFETs operating in the triode region. Clever techniques have been evolved to obtain linear operation with large input signals. Because of space limitations, we shall not study this design method here and refer the reader to Tsividis and Voorman (1992).

EARLY FILTER PIONEERS: CAUER AND DARLINGTON

While on a fellowship with Vannevar Bush at MIT and Harvard, the German mathematician Wilhelm Cauer (1900–1945) used the Chebyshev polynomials in a way that unified the field of filter transfer function design. The elliptical filters now known as Cauer filters have equiripple performance in both the passband and the stopband(s). Cauer continued to make contributions to LC filter synthesis until his mysterious disappearance and presumed death in Berlin on the last day of the Second World War.

Sidney Darlington (1906–1997) developed a complete design theory for LC filters while working at the Bell Telephone Laboratories in the 1940s. Ironically, in later years he became better known for his invention of a particular transistor circuit, the Darlington pair.

Switched-Capacitor Filters These are based on the ingenious technique of obtaining a large resistance by switching a capacitor at a relatively high frequency. Because of the switching action, the resulting filters are discrete-time circuits, as opposed to the continuous-time filters studied thus far. The switched-capacitor approach is ideally suited for implementing low-frequency filters in IC form using CMOS technology. Switched-capacitor filters are studied in Section 14.8 of the eighth edition of the textbook.

x6.4.2 Transconductors

Figure x6.13(a) shows the circuit symbol for a transconductor, and Fig. x6.13(b) shows its equivalent circuit. Here we are assuming the transconductor to be ideal, with infinite input and output impedances. Actual transconductors will obviously deviate from this ideal model. We shall investigate the effects of nonidealities in some of the end-of-chapter problems. Otherwise, we shall assume that for the purpose of this introductory study, the transconductors are ideal.

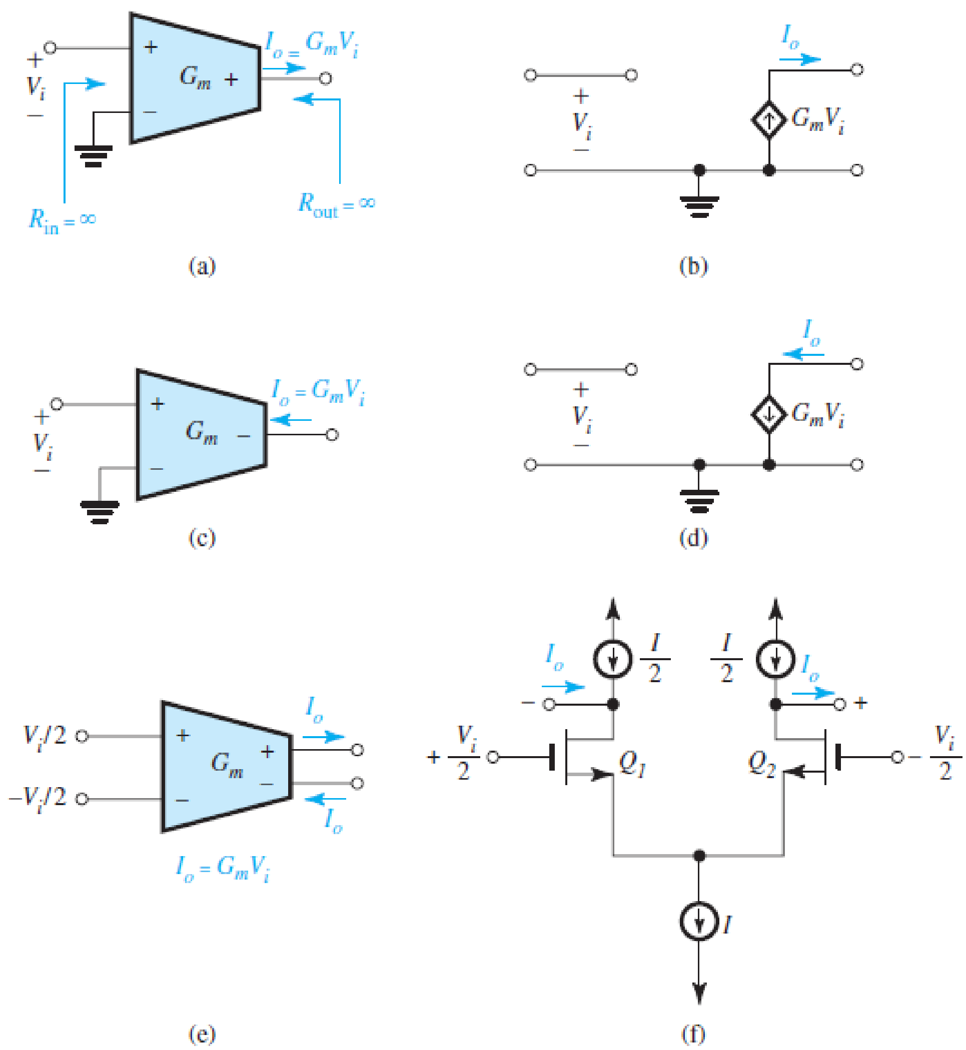


Figure x6.13 **(a)** A positive transconductor; **(b)** equivalent circuit of the transconductor in (a); **(c)** a negative transconductor and its equivalent circuit (d); **(e)** a fully differential transconductor; **(f)** a simple circuit implementation of the fully differential transconductor.

The transconductor of Fig. x6.13(a) has a positive output; that is, the output current $I_o = G_m V_i$ flows *out* of the output terminal. Transconductors with a negative output are, of course, also possible and one is shown in Fig. x6.13(c), with its ideal model in Fig. x6.13(d).

The transconductors of Fig. x6.13(a) and (c) are both of the single-ended type. As mentioned in Chapter 9, differential amplification is preferred over the single-ended variety for a number of reasons, including lower susceptibility to noise and interference. This preference for fully differential operation extends to other signal-processing functions including filtering where it can be shown that distortion, an important issue in filter design, is reduced in fully differential configurations. As a result, at the present time, most IC analog filters utilize fully differential circuits. For this purpose, we show in Fig. x6.13(e) a differential-input–differential-output transconductor.

We have already encountered circuits for implementing transconductors. As an example of a simple implementation, we show the circuit in Fig. x6.13(f), which is simply a differential amplifier loaded with two current sources. The linearity of this circuit is of course limited by the i_D–v_{GS} characteristic of the MOSFET, necessitating the use of small input signals. Many elaborate transconductor circuits have been proposed and utilized in the design of G_m–C filters (see Chan Carusone, Johns, and Martin, 2012).

x6.4.3 Basic Building Blocks

In this section we present the basic building blocks of G_m–C filters. Figure x6.14(a) shows how a negative transconductor can be used to realize a resistance. An integrator is obtained by feeding the output current of a transconductor, $G_m V_i$, to a grounded capacitor, as shown in Fig. x6.14(b). The transfer function obtained is

$$\frac{V_o}{V_i} = \frac{G_m}{sC} \qquad \text{(x6.28)}$$

which is ideal because we have assumed the transconductor to be ideal.

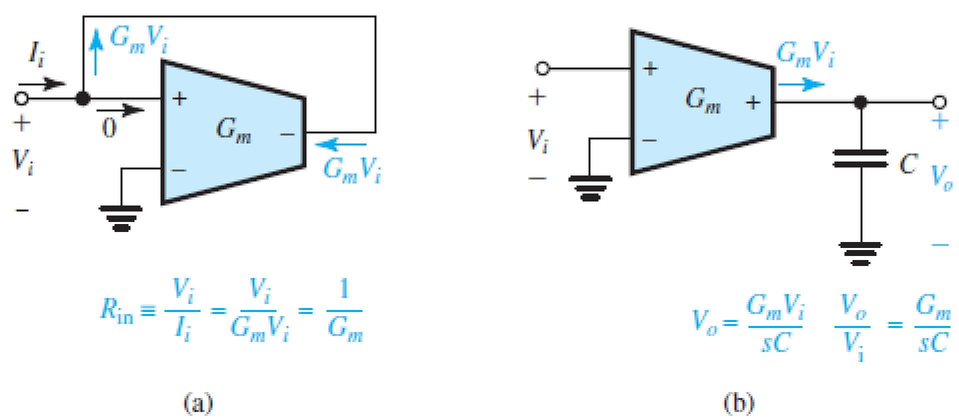


Figure x6.14 Realization of **(a)** a resistance using a negative transconductor; **(b)** an ideal noninverting integrator; **(c)** a first-order low-pass filter (a damped integrator); and **(d)** a fully differential first-order low-pass filter. **(e)** Alternative realization of the fully differential first-order low-pass filter.

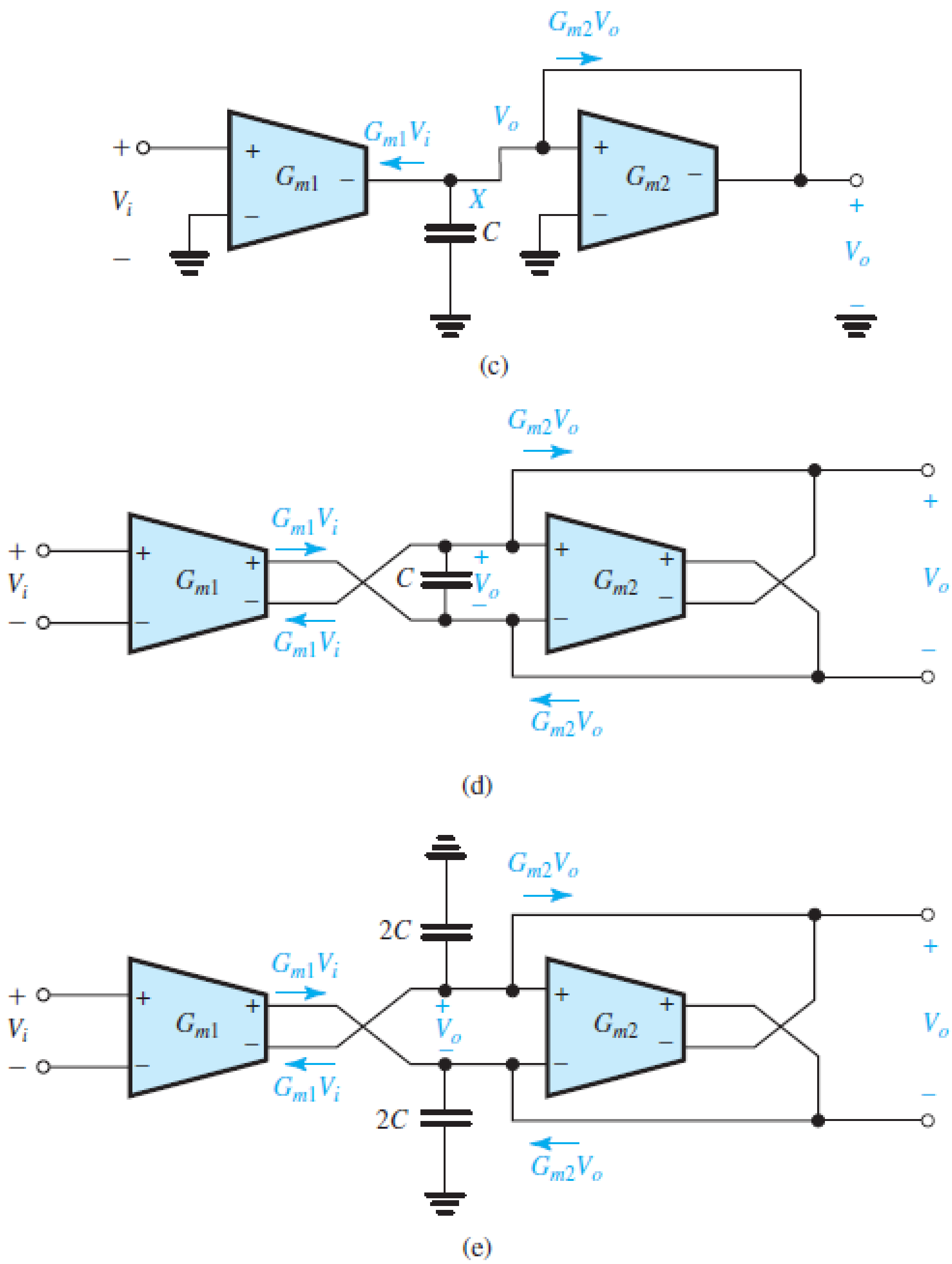


Figure x6.14 *continued*

To obtain a damped integrator, or a first-order low-pass filter, we connect a resistance of the type in Fig. x6.14(a) in parallel with the capacitor C in the integrator of Fig. x6.14(b). The resulting circuit is shown in Fig. x6.14(c). The transfer function can be obtained by writing a node equation at X. The result is

$$\frac{V_o}{V_i} = -\frac{G_{m1}}{sC + G_{m2}} \tag{x6.29}$$

Thus, the pole frequency is (G_{m2}/C) and the dc gain is $(-G_{m1}/G_{m2})$.

The circuit in Figure x6.14(c) can be easily converted to the fully differential form shown in Fig. x6.14(d). An alternative implementation of the fully differential first-order low-pass filter is shown in Fig. x6.14(e). Note that the latter circuit requires four times the capacitance value of the circuit in Fig. x6.14(d). Nevertheless, the circuit of Fig. x6.14(e) has some advantages (see Chan Carusone et al., 2012).

x6.4.4 Second-Order G_m–C Filter

To obtain a second-order G_m–C filter, we use the two-integrator-loop topology of Fig. 14.25(a) in the textbook. Absorbing the ($1/Q$) branch within the first integrator, and lumping the second integrator together with the inverter into a single noninverting integrator block, we obtain the block diagram in Fig. x6.15(a). This block diagram can be easily implemented by G_m−C circuits, resulting in the circuit of Fig. x6.15(b). Note that

1. The inverting integrator is realized by the inverting transconductor G_{m1}, capacitor C_1, and the resistance implemented by transconductor G_{m3}.
2. The noninverting integrator is realized by the noninverting transconductor G_{m2} and capacitor C_2.
3. The input summer is implemented by transconductor G_{m4}, which feeds an output current $G_{m4}V_i$ to the integrator capacitor C_1, and transconductor G_{m1}, which feeds an output current $G_{m1}V_2$ to C_1.

To derive the transfer functions (V_1/V_i) and (V_2/V_i) we first note that V_2 and V_1 are related by

$$V_2 = \frac{G_{m2}}{sC_2}V_1 \tag{x6.30}$$

Next, we write a node equation at X and use the relationship above to eliminate V_2. After some simple algebraic manipulations we obtain

$$\frac{V_1}{V_i} = -\frac{s(G_{m4}/C_1)}{s^2 + s\frac{G_{m3}}{C_1} + \frac{G_{m1}G_{m2}}{C_1C_2}} \tag{x6.31}$$

Now, using Eq. (x6.30) to replace V_1 in Eq. (x6.31) results in

$$\frac{V_2}{V_i} = -\frac{G_{m2}G_{m4}/C_1C_2}{s^2 + s\frac{G_{m3}}{C_1} + \frac{G_{m1}G_{m2}}{C_1C_2}} \tag{x6.32}$$

Thus, the circuit in Fig. x6.15(b) is capable of realizing simultaneously a bandpass function (V_1/V_i) and a low-pass function (V_2/V_i). For both

$$\omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}} \tag{x6.33}$$

and

$$Q = \frac{\sqrt{G_{m1}G_{m2}}}{G_{m3}}\sqrt{\frac{C_1}{C_2}} \tag{x6.34}$$

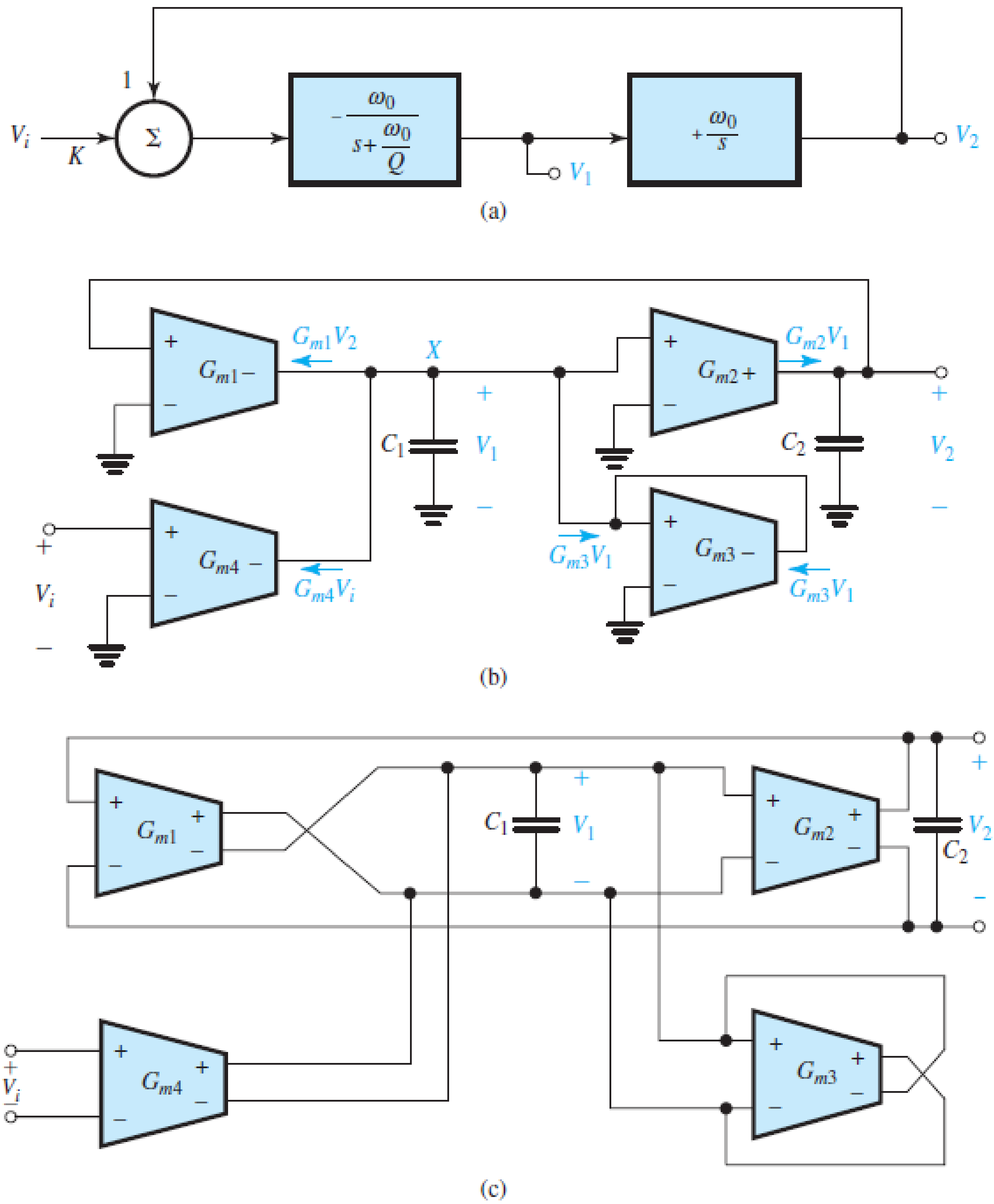


Figure x6.15 **(a)** Block diagram of the two-integrator-loop biquad. This is a somewhat modified version of Fig. 14.25 in the textbook. **(b)** G_m–C implementation of the block diagram in (a). **(c)** Fully differential G_m–C implementation of the block diagram in (a). In all parts, V_1/V_i is a bandpass function and V_2/V_i is a low-pass function.

For the bandpass function,

$$\text{Center frequency gain} = -\frac{G_{m4}}{G_{m3}} \quad \text{(x6.35)}$$

and for the low-pass function,

$$\text{DC gain} = -\frac{G_{m4}}{G_{m1}} \quad \text{(x6.36)}$$

There are a variety of possible designs. The most common is to make the time constants of the integrators equal [which is the case in the block diagram of Fig. x6.15(a)]. Doing this and selecting $G_{m1} = G_{m2} = G_m$ and $C_1 = C_2 = C$ results in the following design equation:

$$\frac{G_m}{C} = \omega_0 \quad \text{(x6.37)}$$

$$G_{m3} = \frac{G_m}{Q} \quad \text{(x6.38)}$$

$$\text{For the BP: } G_{m4} = \frac{G_m}{Q}|\text{Gain}| \quad \text{(x6.39)}$$

$$\text{For the LP: } G_m = G_m|\text{Gain}| \quad \text{(x6.40)}$$

Example x6.2

Design the G_m–C circuit of Fig. x6.15(b) to realize a bandpass filter with a center frequency of 10 MHz, a 3-dB bandwidth of 1 MHz, and a center-frequency gain of 10. Use equal capacitors of 5 pF.

Solution

Using the equal-integrator-time-constants design, Eq. (x6.37) yields

$$G_m = \omega_0 C = 2\pi \times 10 \times 10^6 \times 5 \times 10^{-12} = 0.314 \text{ mA/V}$$

Thus,

$$G_{m1} = G_{m2} = 0.314 \text{ mA/V}$$

To obtain G_{m3}, we first note that $Q = f_0/\text{BW} = 10/1 = 10$, and then use Eq. (x6.38) to obtain

$$G_{m3} = \frac{G_m}{Q} = \frac{0.314}{10} = 0.0314 \text{ mA/V}$$

or

$$Gm3 = 31.4\ \mu A/V$$

Finally, G_{m4} can be found by using Eq. (x6.39) as

$$G_{m4} = \frac{G_m}{10} \times 10 = 0.314\ \text{mA/V}$$

We note that the feedforward approach used in Section 14.6.3 of the textbook to realize different transmission zeros (as required for high-pass, notch, and all-pass functions) can be adapted to the G_m–C circuit in Fig. x6.15(b). Some of these possibilities are explored in the end-of-chapter problems. Finally, the circuit in Fig. x6.15(b) can be easily converted to the fully differential form shown in Fig. x6.15(c).

EXERCISE

xD6.13 Design the circuit of Fig. x6.15(b) to realize a maximally flat low-pass filter with f_{3dB} = 20 MHz and a dc gain of unity. Design for equal integrator time constants, and use equal capacitors of 2 pF each.

Ans. $G_{m1} = G_{m2} = G_{m4} = 0.251$ mA/V; $G_{m3} = 0.355$ mA/V

This concludes our study of G_m–C filters. The interested reader can find considerably more material on this subject in Schaumann et al. (2010).

x6.5 Tuned Amplifiers

We conclude this supplement with the study of a special kind of frequency-selective network, the LC-tuned amplifier. Figure x6.16 shows the general shape of the frequency response of a tuned amplifier. The techniques discussed apply to amplifiers with center frequencies in the range of a few hundred kilohertz to a few hundred megahertz. Tuned amplifiers find application in the radio-frequency (RF) and intermediate-frequency (IF) sections of communications receivers and in a variety of other systems. It should be noted that the tuned-amplifier response of Fig. x6.16 is similar to that of the bandpass filter discussed in earlier sections.

As indicated in Fig. x6.16, the response is characterized by the center frequency ω_0, the 3-dB bandwidth B, and the *skirt selectivity,* which is usually measured as the ratio of the 30-dB bandwidth to the 3-dB bandwidth. In many applications, the 3-dB bandwidth is less than 1% of ω_0. This **narrow-band** property makes possible certain approximations that can simplify the design process.

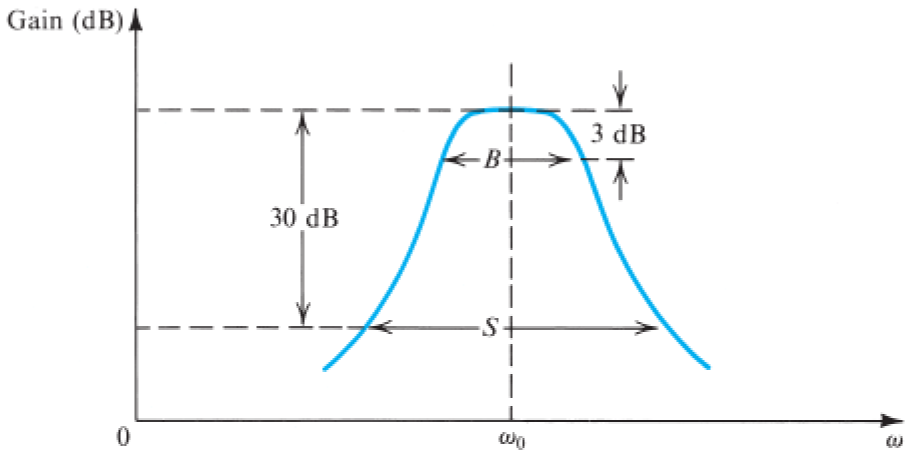


Figure x6.16 Frequency response of a tuned amplifier.

The tuned amplifiers discussed in this section can be implemented in discrete-circuit form using transistors together with passive inductors and capacitors. Increasingly, however, they are implemented in IC form, where the inductors are specially fabricated by depositing thin metal films in a spiral shape. These IC inductors, however, are very small and hence are useful only in very-high-frequency applications. Also they usually have considerable losses or, equivalently, low Q factors. Various circuit techniques have been proposed to raise the realized Q factors. These usually involve an amplifier circuit that generates a negative resistance, which is connected to the inductor in a way that cancels part of its resistance and thus enhance its Q factor. The resulting tuned amplifiers are therefore referred to as *active-LC filters* (see Schaumann et al., 2010).

This section considers tuned amplifiers that are small-signal voltage amplifiers in which the transistors operate in the "class A" mode; that is, the transistors conduct at all times. Tuned power amplifiers such as those based on class C operation of the transistor, are not studied in this book. (For a discussion on the classification of amplifiers, refer to Section 12.1.)

x6.5.1 The Basic Principle

The basic principle underlying the design of tuned amplifiers is the use of a parallel LCR circuit as the load, or at the input, of a BJT or an FET amplifier. This is illustrated in Fig. x6.17 with a MOSFET amplifier having a tuned-circuit load. For simplicity, the bias details are not included. Since this circuit uses a single tuned circuit, it is known as a **single-tuned amplifier**. The amplifier equivalent circuit is shown in Fig. x6.17(b). Here R denotes the parallel equivalent of R_L and the output resistance r_o of the FET, and C is the parallel equivalent of C_L and the FET output capacitance (usually small). From the equivalent circuit we can write

$$V_o \equiv \frac{-g_m V_i}{Y_L} = \frac{-g_m V_i}{sC + 1/R + 1/sL}$$

Thus the voltage gain can be expressed as

$$\frac{V_o}{V_i} = -\frac{g_m}{C}\frac{s}{s^2 + s(1/CR) + 1/LC} \qquad \text{(x6.41)}$$

which is a second-order bandpass function. Thus the tuned amplifier has a center frequency of

$$\omega_0 = 1/\sqrt{LC} \tag{x6.42}$$

a 3-dB bandwidth of

$$B = \frac{1}{CR} \tag{x6.43}$$

a Q factor of

$$Q \equiv \omega_0/B = \omega_0 CR \tag{x6.44}$$

and a center-frequency gain of

$$\frac{V_o(j\omega_0)}{V_i(j\omega_0)} = -g_m R \tag{x6.45}$$

Note that the expression for the center-frequency gain could have been written by inspection; at resonance, the reactances of L and C cancel out and the impedance of the parallel LCR circuit reduces to R.

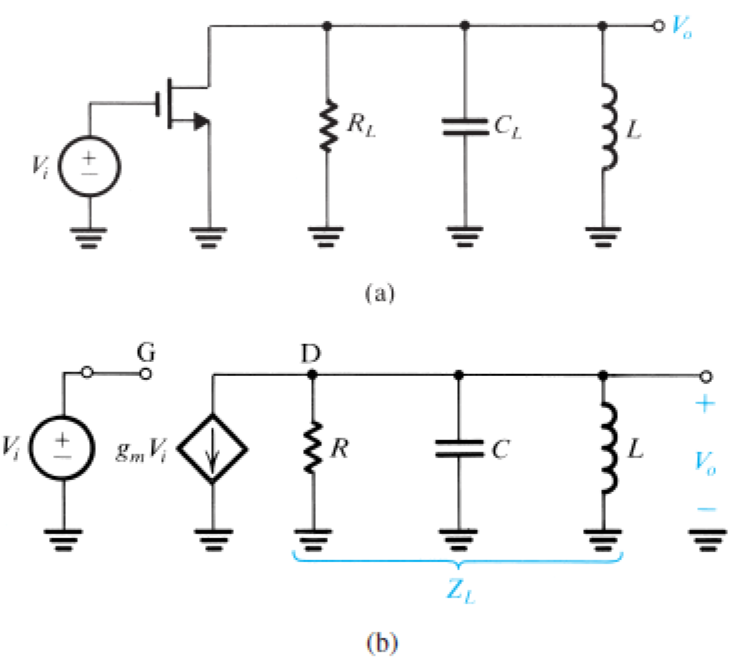


Figure x6.17 The basic principle of tuned amplifiers is illustrated using a MOSFET with a tuned-circuit load. Bias details are not shown.

Example x6.3

We are required to design a tuned amplifier of the type shown in Fig. x6.17, having f_0 = 1 MHz, 3-dB bandwidth = 10 kHz, and center-frequency gain = –10 V/V. The FET available has at the bias point g_m = 5 mA/V and r_o = 10 kΩ. The output capacitance is negligibly small. Determine the values of R_L, C_L, and L.

Solution

Center-frequency gain = −10 = −5R. Thus R = 2 kΩ. Since $R = R_L \parallel r_o$, then R_L = 2.5 kΩ.

$$B = 2\pi \times 10^4 = \frac{1}{CR}$$

Thus

$$C = \frac{1}{2\pi \times 10^4 \times 2 \times 10^3} = 7958 \text{ pF}$$

Since $\omega_0 = 2\pi \times 10^6 = 1/\sqrt{LC}$, we obtain

$$L = \frac{1}{4\pi^2 \times 10^{12} \times 7958 \times 10^{-12}} = 3.18 \text{ μH}$$

x6.5.2 Inductor Losses

The power loss in the inductor is usually represented by a series resistance r_s as shown in Fig. x6.18(a). However, rather than specifying the value of r_s, the usual practice is to specify the inductor Q factor at the frequency of interest,

$$Q_0 \equiv \frac{\omega_0 L}{r_s} \tag{x6.46}$$

Typically, Q_0 is in the range of 50 to 200.

The analysis of a tuned amplifier is greatly simplified by representing the inductor loss by a parallel resistance R_p, as shown in Fig. x6.18(b). The relationship between R_p and Q_0 can be found by writing, for the admittance of the circuit in Fig. x6.18(a),

$$Y(j\omega_0) = \frac{1}{r_s + j\omega_0 L}$$
$$= \frac{1}{j\omega_0 L}\,\frac{1}{1 - j(1/Q_0)}$$
$$= \frac{1}{j\omega_0 L}\,\frac{1 + j(1/Q_0)}{1 + (1/Q_0^2)}$$

For $Q_0 \gg 1$,

$$Y(j\omega_0) \simeq \frac{1}{j\omega_0 L}\left(1 + j\frac{1}{Q_0}\right) \tag{x6.47}$$

Equating this to the admittance of the circuit in Fig. x6.18(b) gives

$$Q_0 = \frac{R_p}{\omega_0 L} \tag{x6.48}$$

or, equivalently,

$$R_p = \omega_0 L Q_0 \tag{x6.49}$$

Finally, it should be noted that the coil Q factor poses an upper limit on the value of Q achieved by the tuned circuit.

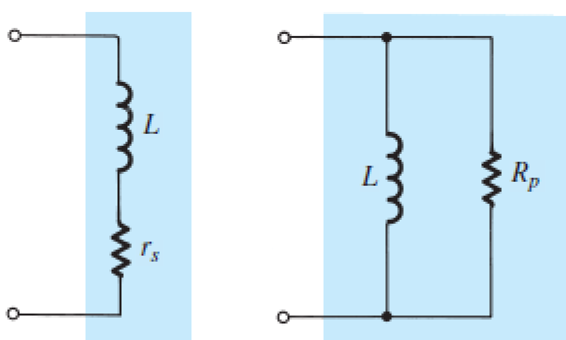


Figure x6.18 Inductor equivalent circuits.

EXERCISE

x6.14 If the inductor in Example x6.3 has Q_0 = 150, find R_p and then find the value to which R_L should be changed to keep the overall Q, and hence the bandwidth, unchanged.

Ans. 3 kΩ; 15 kΩ

x6.5.3 Use of Transformers

In many cases it is found that the required value of inductance is not practical, in the sense that coils with the required inductance might not be available with the required high values ofQ_0. A simple solution is to use a transformer to effect an impedance change. Alternatively, a tapped coil, known as an **autotransformer**, can be used, as shown in Fig. x6.19. Provided the two parts of the inductor are tightly coupled, which can be achieved by winding on a ferrite core, the transformation relationships shown hold. The result is that the tuned circuit seen between terminals 1 and $1'$ is equivalent to that in Fig. x6.17(b). For example, if a turns ratio $n = 3$ is used in the amplifier of Example x6.3, then a coil with inductance $L' = 9 \times 3.18 = 28.6\ \mu H$ and a capacitance $C' = 7958/9 = 884$ pF will be required. Both these values are more practical than the original ones.

In applications that involve coupling the output of a tuned amplifier to the input of another amplifier, the tapped coil can be used to raise the effective input resistance of the latter amplifier stage. In this way, one can avoid reduction of the overall Q. This point is illustrated in Fig. x6.20 and in the following exercises.

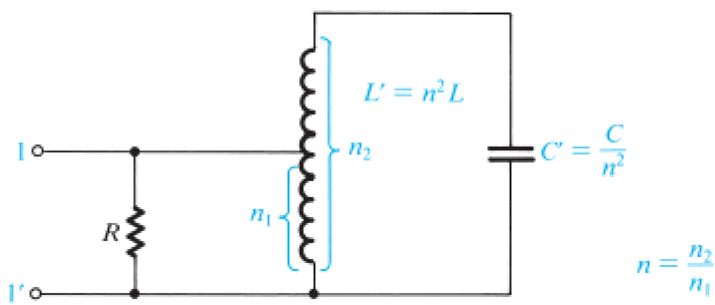


Figure x6.19 A tapped inductor is used as an impedance transformer to allow using a higher inductance, L', and a smaller capacitance, C'.

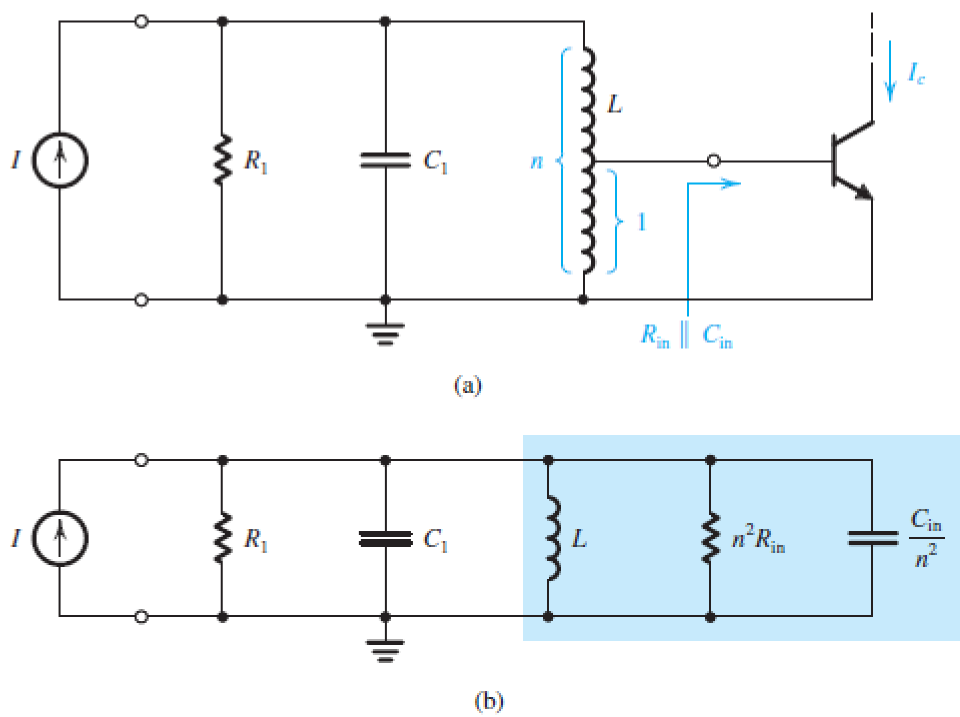


Figure x6.20 **(a)** The output of a tuned amplifier is coupled to the input of another amplifier via a tapped coil. **(b)** An equivalent circuit. Note that the use of a tapped coil increases the effective input impedance of the second amplifier stage.

EXERCISES

x6.15 Consider the circuit in Fig. x6.20(a), first without tapping the coil. Let $L = 5\ \mu$H and assume that R_1 is fixed at 1 kΩ. We wish to design a tuned amplifier with $f_0 = 455$ kHz and a 3-dB bandwidth of 10 kHz [this is the intermediate frequency (IF) amplifier of an AM radio]. If the BJT has $R_{in} = 1$ kΩ and $C_{in} = 200$ pF, find the actual bandwidth obtained and the required value of C_1.

Ans. 13 kHz; 24.27 nF

xD6.16 Since the bandwidth realized in Exercise x6.15 is greater than desired, find an alternative design utilizing a tapped coil as in Fig. x6.20(a). Find the value of n that allows the specifications to be just met. Also find the new required value of C_1 and the current gain I_c/I at resonance. Assume that at the bias point the BJT has $g_m = 40$ mA/V.

Ans. 1.36; 24.36 nF; 19.1 A/A

x6.5.4 Amplifiers with Multiple Tuned Circuits

The selectivity achieved with the single tuned circuit of Fig. x6.17 is not sufficient in many applications—for instance, in the IF amplifier of a radio or a TV receiver. Greater selectivity is obtained by using additional tuned stages. Figure x6.21 shows a BJT with tuned circuits at both the input and the output. In this circuit the bias details are shown, from which we note that biasing is quite similar to the classical arrangement employed in low-frequency, discrete-circuit design. However, to avoid the loading effect of the bias resistors R_{B1} and R_{B2} on the input tuned circuit, a **radio-frequency choke** (RFC) is inserted in series with each resistor. Such chokes have low resistance but high impedances at the frequencies of interest. The use of RFCs in biasing tuned RF amplifiers is common practice.

The analysis and design of the double-tuned amplifier of Fig. x6.21 is complicated by the Miller effect due to capacitance C_μ. Since the load is not simply resistive, as is the case in the amplifiers studied in Section 10.2.4 of the textbook, the Miller impedance at the input will be complex. This reflected impedance will cause detuning of the input circuit as well as "skewing" of the response of the input circuit. Needless to say, the coupling introduced by C_μ makes tuning (or aligning) the amplifier quite difficult. Worse still, the capacitor C_μ can cause oscillations to occur (see Gray and Searle, 1969).

Methods exist for **neutralizing** the effect of C_μ, using additional circuits arranged to feed back a current equal and opposite to that through C_μ. An alternative, and preferred, approach is to use circuit configurations that do not suffer from the Miller effect. These are discussed later. Before leaving this section, however, we wish to point out that circuits of the type shown in Fig. x6.21 are usually designed utilizing the y-parameter model of the BJT (see Appendix C). This is done because here, in view of the fact that C_μ plays a significant role, the y-parameter model makes the analysis simpler (in comparison to that using the hybrid-π model). Also, the y parameters can easily be measured at the particular frequency of interest, ω_0. For narrow-band amplifiers, the assumption is usually made that the y parameters remain approximately constant over the passband.

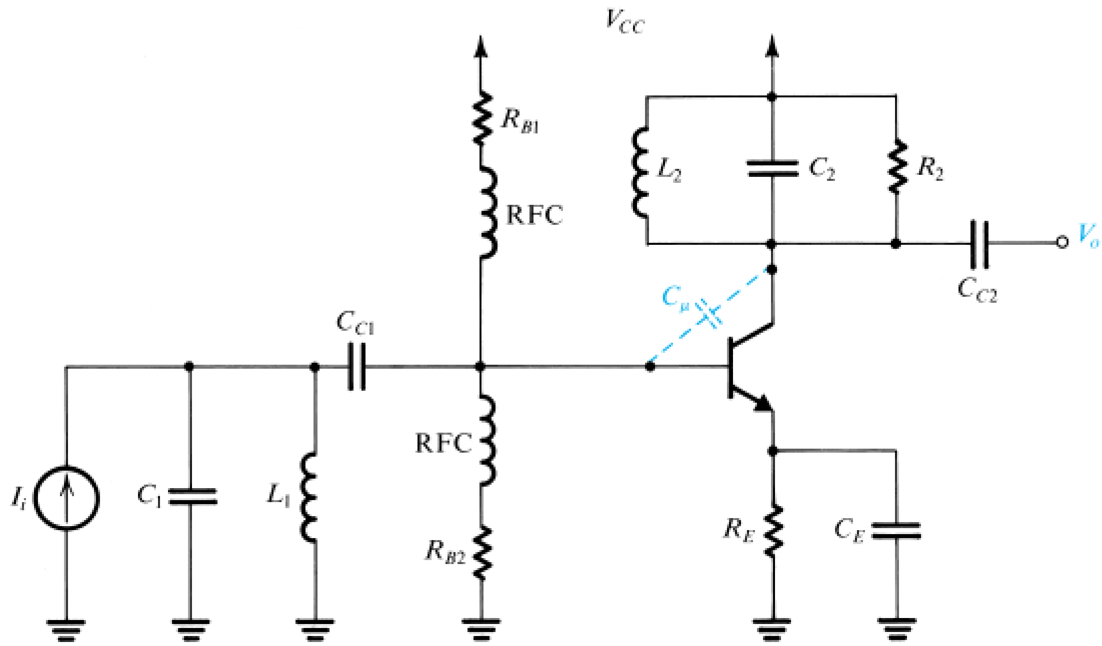


Figure x6.21 A BJT amplifier with tuned circuits at the input and the output.

x6.5.5 The Cascode and the CC–CB Cascade

From our study of amplifier frequency response in Chapter 10, we know that two amplifier configurations do not suffer from the Miller effect. These are the cascode configuration and the common-collector, common-base cascade. Figure x6.22 shows tuned amplifiers based on these two configurations. The CC–CB cascade is usually preferred in IC implementations because its differential structure makes it suitable for IC biasing techniques. (Note that the biasing details of the cascode circuit are not shown in Fig. x6.22(a). Biasing can be done using arrangements similar to those discussed in earlier chapters.)

x6.5.6 Synchronous Tuning and Stagger Tuning

In the design of a tuned amplifier with multiple tuned circuits, the question of the frequency to which each circuit should be tuned arises. The objective, of course, is for the overall response to exhibit high passband flatness and skirt selectivity. To investigate this question, we shall assume that the overall response is the product of the individual responses: in other words, that the stages do not interact. This can easily be achieved using circuits such as those in Fig. x6.22.

Consider first the case of N identical resonant circuits, known as the **synchronously tuned** case. Figure x6.23 shows the response of an individual stage and that of the cascade. Observe the bandwidth "shrinkage" of the overall response. The 3-dB bandwidth B of the overall amplifier is related to that of the individual tuned circuits, ω_0/Q, by

$$B = \frac{\omega_0}{Q}\sqrt{2^{1/N} - 1} \quad \text{(x6.50)}$$

The factor $\sqrt{2^{1/N} - 1}$ is known as the **bandwidth-shrinkage factor**. Given B and N, we can se Eq. (x6.50) to determine the bandwidth required of the individual stages ω_0/Q.

EXERCISE

x6.18 Consider the design of an IF amplifier for an FM radio receiver. Using two synchronously tuned stages with $f_0 = 10.7$ MHz, find the 3-dB bandwidth of each stage so that the overall bandwidth is 200 kHz. Using 3-μH inductors find C and R for each stage.

Ans. 310.8 kHz; 73.7 pF; 6.95 kΩ

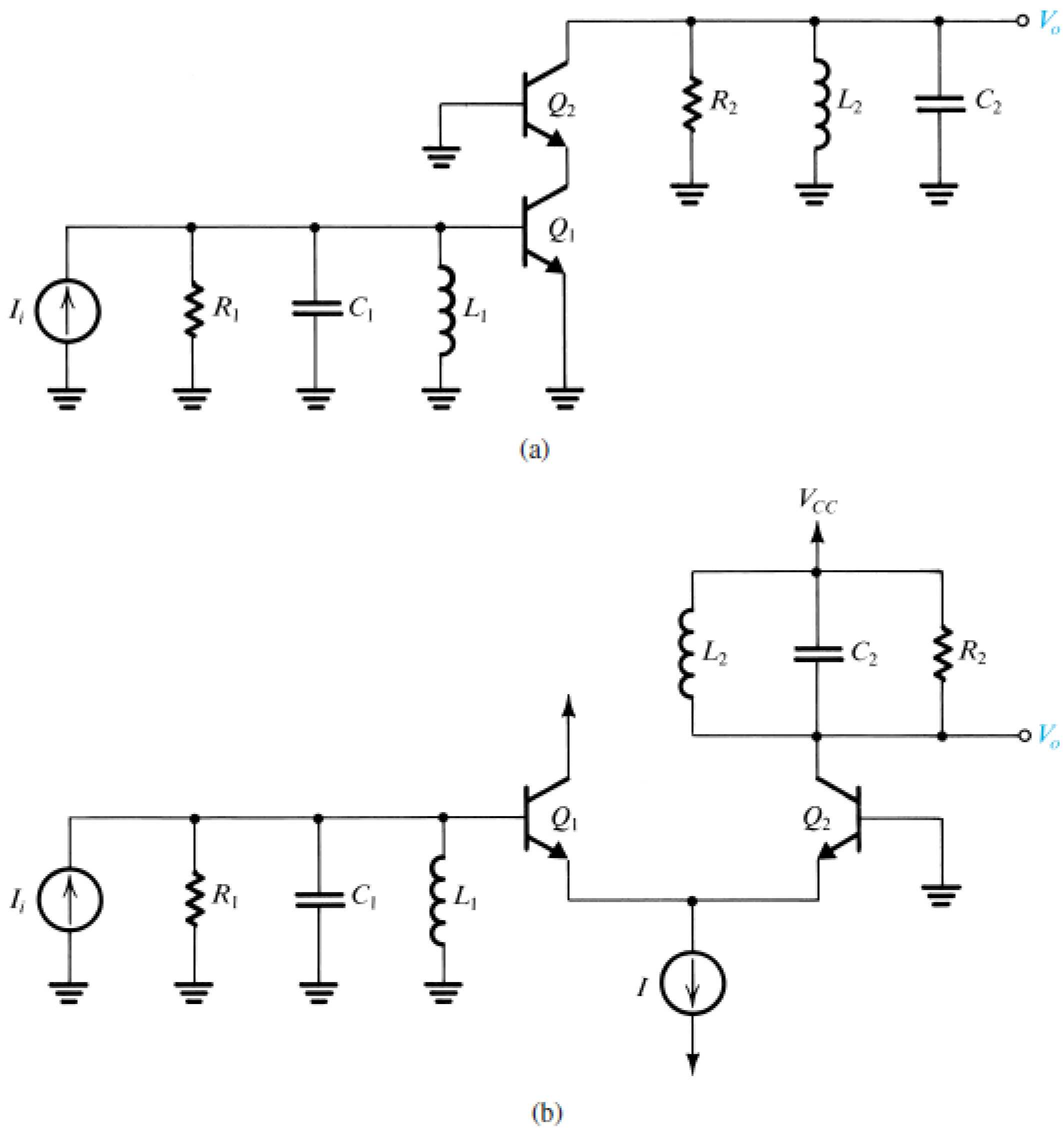


Figure x6.22 Two tuned-amplifier configurations that do not suffer from the Miller effect: **(a)** cascode and **(b)** common-collector, common-base cascade. (Note that bias details of the cascode circuit are not shown.)

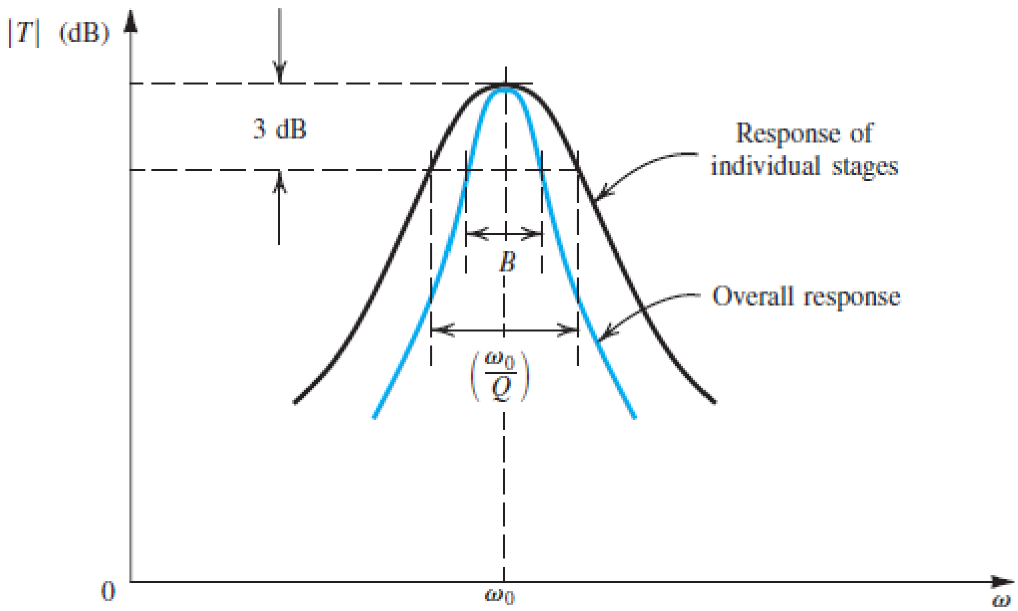


Figure x6.23 Frequency response of a synchronously tuned amplifier.

A much better overall response is obtained by stagger-tuning the individual stages, as illustrated in Fig. x6.24. Stagger-tuned amplifiers are usually designed so that the overall response exhibits *maximal flatness* around the center frequency f_0. Such a response can be obtained by transforming the response of a maximally flat (Butterworth) low-pass filter up the frequency axis to ω_0.

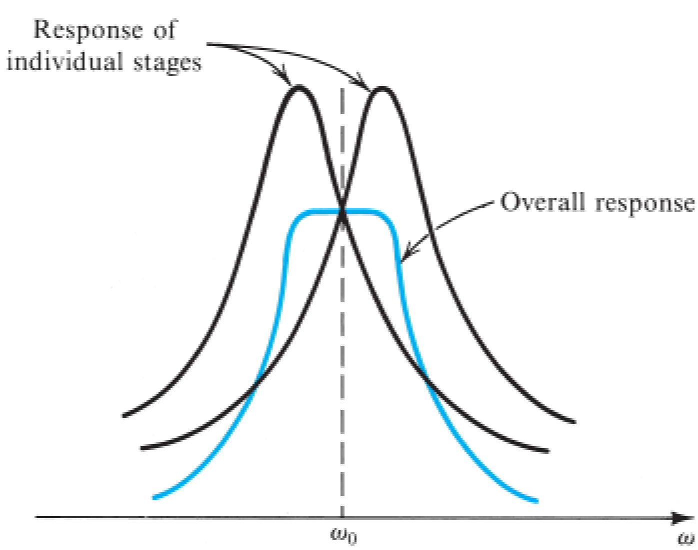


Figure x6.24 Stagger-tuning the individual resonant circuits can result in an overall response with a passband flatter than that obtained with synchronous tuning (Fig. x6.23).