**APPENDIX C**

# TWO-PORT NETWORK PARAMETERS

## Introduction

At various points throughout the text, we make use of some of the different possible ways to characterize linear two-port networks. A summary of this topic is presented in this appendix.

## C.1 Characterization of Linear Two-Port Networks

A two-port network (Fig. C.1) has four port variables: $V_1$, $I_1$,$V_2$, and $I_2$. If the two-port network is linear, we can use two of the variables as excitation variables and the other two as response variables. For instance, the network can be excited by a voltage $V_1$ at port 1 and a voltage $V_2$ at port 2, and the two currents, $I_1$ and $I_2$, can be measured to represent the network response. In this case, $V_1$ and $V_2$ are independent variables and $I_1$ and $I_2$ are dependent variables, and the network operation can be described by the two equations

$$I_1 = y_{11}V_1 + y_{12}V_2 \tag{C.1}$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \tag{C.2}$$

Here, the four parameters $y_{11}$, $y_{12}$, $y_{21}$, and $y_{22}$ are admittances, and their values completely characterize the linear two-port network.

Depending on which two of the four port variables are used to represent the network excitation, a different set of equations (and a correspondingly different set of parameters) is obtained for characterizing the network. We shall present the four parameter sets commonly used in electronics.

### C.1.1 *y* Parameters

The short-circuit admittance (or *y*-parameter) characterization is based on exciting the network by $V_1$ and $V_2$, as shown in Fig. C.2(a). The describing equations are Eqs. (C.1) and (C.2). The four admittance parameters can be defined according to their roles in Eqs. (C.1) and (C.2).

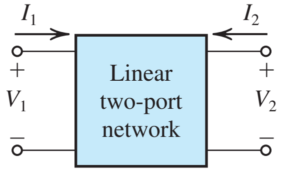


**Figure C.1** The reference directions of the four port variables in a linear two-port network.

Specifically, from Eq. (C.1) we see that $y_{11}$ is defined as

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \tag{C.3}$$

Thus $y_{11}$ is the input admittance at port 1 with port 2 short-circuited. This definition is illustrated in Fig. C.2(b), which also provides a conceptual method for measuring the input short-circuit admittance $y_{11}$.

The definition of $y_{12}$ can be obtained from Eq. (C.1) as

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \tag{C.4}$$

Thus $y_{12}$ represents transmission from port 2 to port 1. Since in amplifiers, port 1 represents the input port and port 2 the output port, $y_{12}$ represents internal *feedback* in the network. Figure C.2(c) illustrates the definition of and the method for measuring $y_{12}$.

The definition of $y_{21}$ can be obtained from Eq. (C.2) as

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \tag{C.5}$$

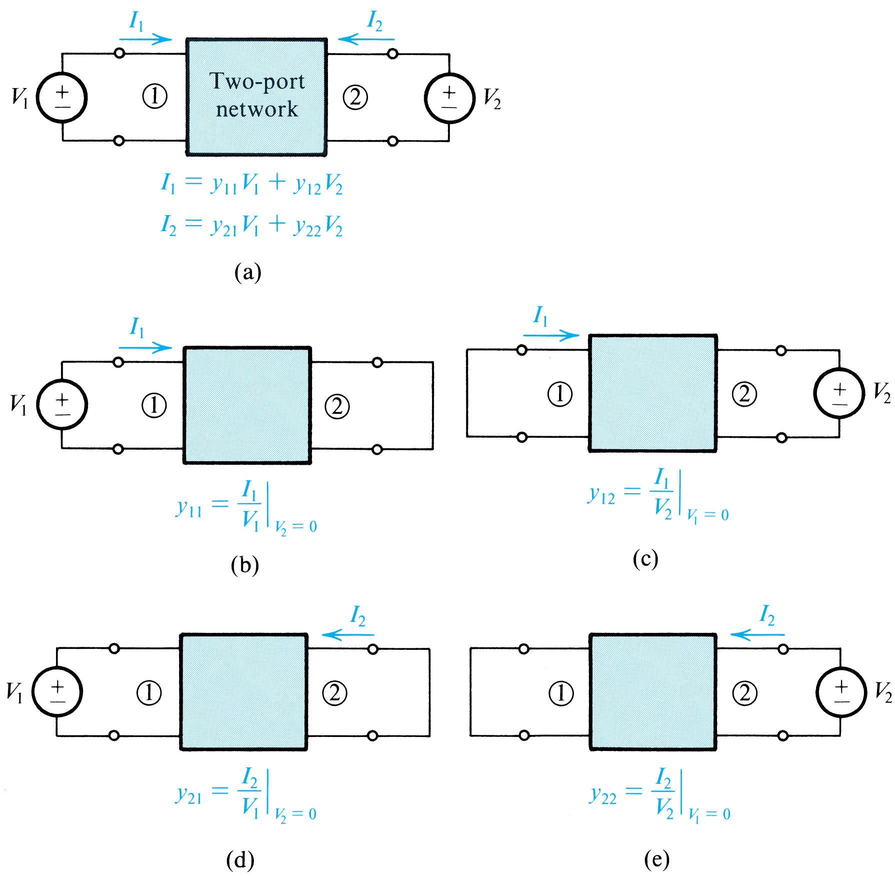


**Figure C.2** Definition of and conceptual measurement circuits for the $y$ parameters.

Thus $y_{21}$ represents transmission from port 1 to port 2. If port 1 is the input port and port 2 the output port of an amplifier, then $y_{21}$ provides a measure of the forward gain or transmission. Figure C.2(d) illustrates the definition of and the method for measuring $y_{21}$.

The parameter $y_{22}$ can be defined, based on Eq. (C.2), as

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \tag{C.6}$$

Thus $y_{22}$ is the admittance looking into port 2 while port 1 is short-circuited. For amplifiers, $y_{22}$ is the output short-circuit admittance. Figure C.2(e) illustrates the definition of and the method for measuring $y_{22}$.

## C.1.2 *z*

The open-circuit impedance (or $z$-parameter) characterization of two-port networks is based on exciting the network by $I_1$ and $I_2$, as shown in Fig. C.3(a). The describing equations are

$$V_1 = z_{11}I_1 + z_{12}I_2 \tag{C.7}$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \tag{C.8}$$

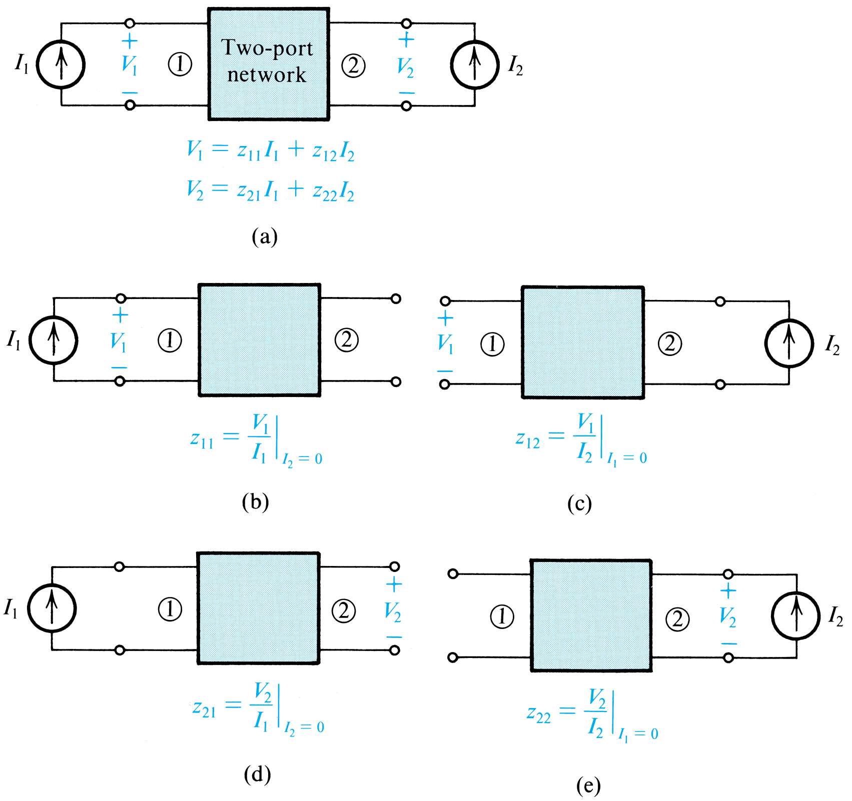


**Figure C.3** Definition of and conceptual measurement circuits for the $z$ parameters.

Owing to the duality between the $z$- and $y$-parameter characterizations, we shall not give a detailed discussion of $z$ parameters. The definition and the method of measuring each of the four $z$ parameters are given in Fig. C.3.

## C.1.3 *h*

The hybrid (or $h$-parameter) characterization of two-port networks is based on exciting the network by $I_1$ and $V_2$, as shown in Fig. C.4(a) (note the reason behind the name *hybrid*). The describing equations are

$$V_1 = h_{11}I_1 + h_{12}V_2 \tag{C.9}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \tag{C.10}$$

from which the definition of the four $h$ parameters can be obtained as

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_0=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

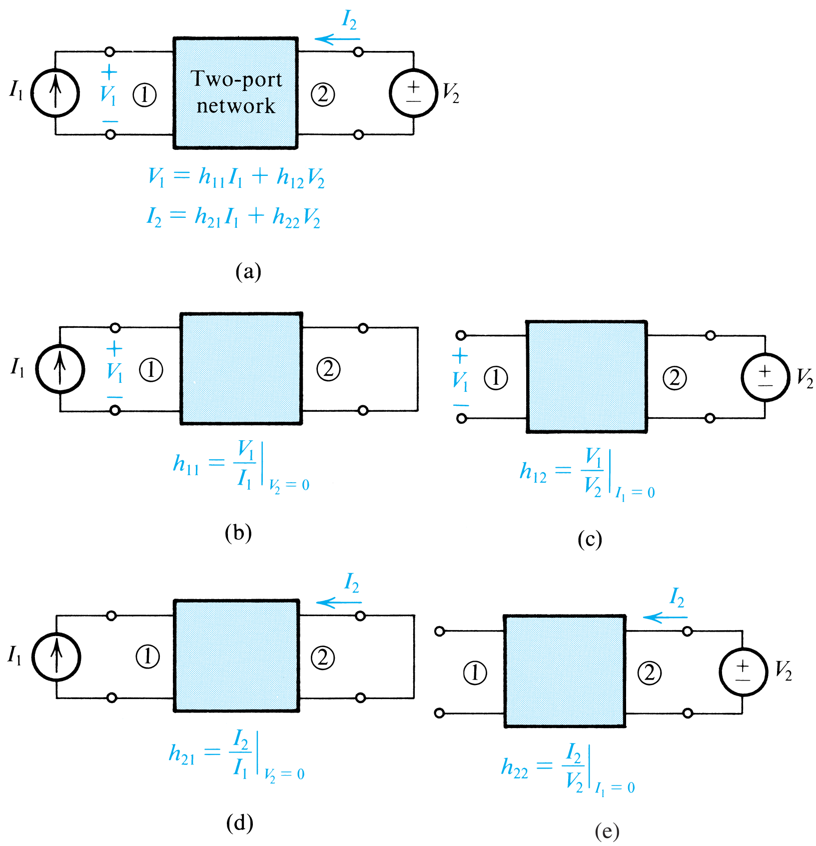


**Figure C.4** Definition of and conceptual measurement circuits for the $h$ parameters.

Thus, $h_{11}$ is the input impedance at port 1 with port 2 short-circuited. The parameter $h_{12}$ represents the reverse or feedback voltage ratio of the network, measured with the input port open-circuited. The forward-transmission parameter $h_{21}$ represents the current gain of the network with the output port short-circuited; for this reason, $h_{21}$ is called the *short-circuit current gain*. Finally, $h_{22}$ is the output admittance with the input port open-circuited.

The definitions and conceptual measuring setups of the $h$ parameters are given in Fig. C.4.

## C.1.4 *g*

The inverse-hybrid (or $g$-parameter) characterization of two-port networks is based on excitation of the network by $V_1$ and $I_2$, as shown in Fig. C.5(a). The describing equations are

$$I_1 = g_{11}V_1 + g_{12}I_2 \tag{C.11}$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \tag{C.12}$$

The definitions and conceptual measuring setups are given in Fig. C.5.

## C.1.5 Equivalent-Circuit Representation

A two-port network can be represented by an equivalent circuit based on the set of parameters used for its characterization. Figure C.6 shows four possible equivalent circuits

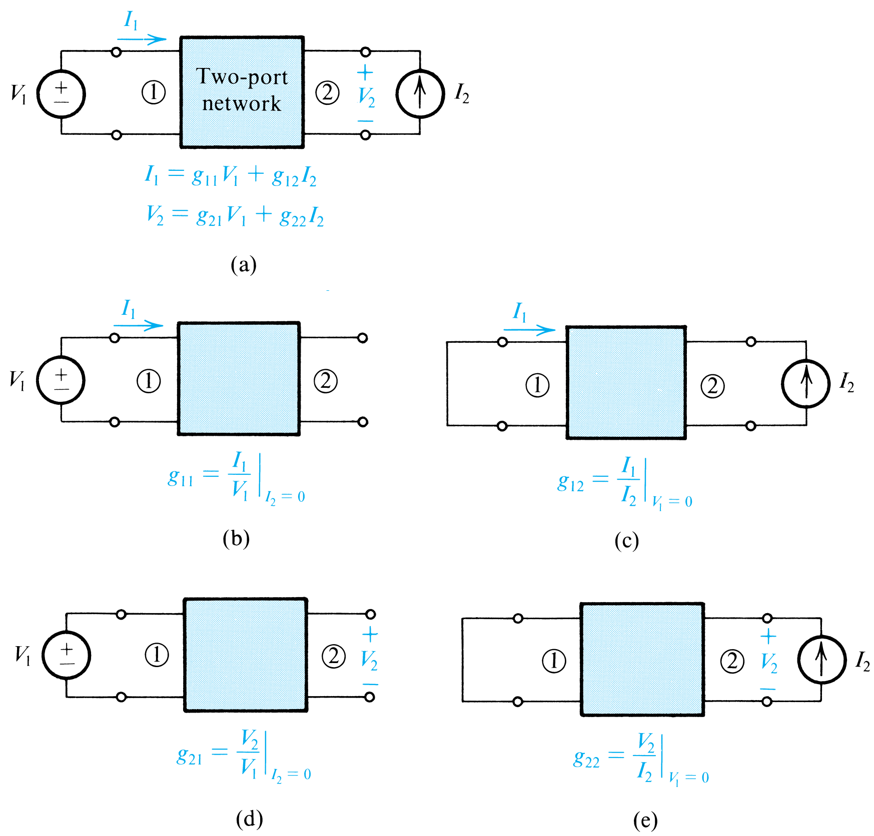


**Figure C.5** Definition of and conceptual measurement circuits for the $g$ parameters.

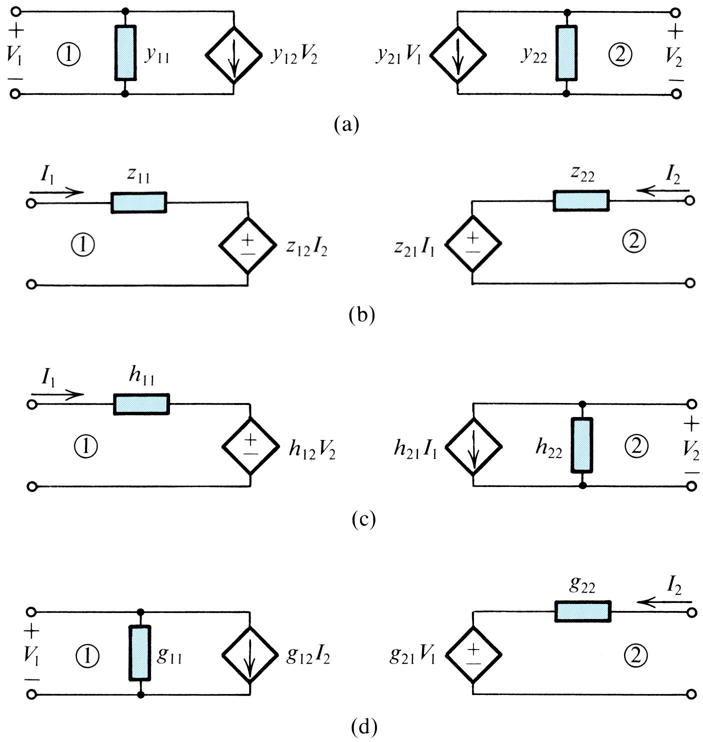


**Figure C.6** Equivalent circuits for two-port networks in terms of (**a**) $y$, (**b**) $z$, (**c**) $h$, and (**d**) $g$ parameters.

corresponding to the four parameter types just discussed. Each of these equivalent circuits is a direct pictorial representation of the corresponding two equations describing the network in terms of the particular parameter set.

Finally, it should be mentioned that other parameter sets exist for characterizing two-port networks, but these are not discussed or used in this book.

## EXERCISES

**C.1** Figure EC.1 shows the small-signal, equivalent-circuit model of a transistor. Calculate the values of the $h$ parameters.

**Ans.** $h_{11} \simeq 2.6\ \text{k}\Omega; h_{12} \simeq 2.5 \times 10^{-4}; h_{21} \simeq 100; h_{22} \simeq 2 \times 10^{-5}\ \Omega$

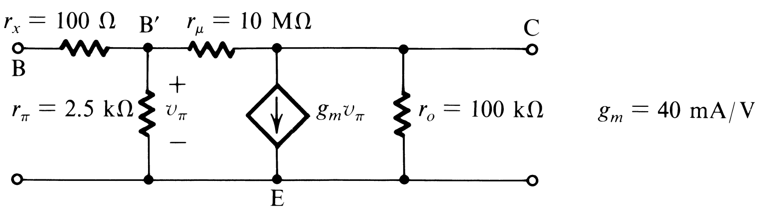


**Figure EC.1**

**C.1** (a) An amplifier characterized by the $h$-parameter equi-valent circuit of Fig. C.6(c) is fed with a source having a voltage $V_s$ and a resistance $R_s$, and is loaded in a resistance $R_L$. Show that its voltage gain is given by

$$\frac{V_2}{V_s} = \frac{-h_{21}}{(h_{11}+R_s)(h_{22}+1/R_L)-h_{12}h_{21}}$$

(b) Use the expression derived in (a) to find the voltage gain of the transistor in Exercise C.1 for $R_s = 1\ \text{k}\Omega$ and $R_L = 10\ \text{k}\Omega$.

**C.2** The terminal properties of a two-port network are measured with the following results: With the output short-circuited and an input current of 0.01 mA, the output current is 1.0 mA and the input voltage is 26 mV. With the input open-circuited and a voltage of 10 V applied to the output, the current in the output is 0.2 mA and the voltage measured at the input is 2.5 mV. Find values for the $h$ parameters of this network.

**C.3** Figure PC.3 shows the high-frequency equivalent circuit of a BJT. (For simplicity, $r_x$ has been omitted.) Find the $y$ parameters.

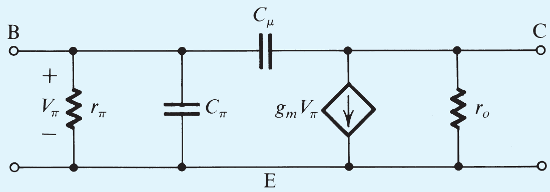


**Figure PC.3**